

# Distributional Methods for Heuristic Search

Tianyi Gu

Advisor: Wheeler Ruml



**University of New Hampshire**

# About Me

---

Introduction

■ About Me

■ Real-time Search

Nancy

Conclusions

6th year CS PhD at UNH

Research: heuristic search and planning

real-time heuristic search

suboptimal search

metareasoning

heuristic search can benefit from representing uncertainty

scalar heuristic → belief distribution that represents uncertainty

# About Me

---

Introduction

■ About Me

■ Real-time Search

Nancy

Conclusions

6th year CS PhD at UNH

Research: heuristic search and planning

**real-time heuristic search**

suboptimal search

metareasoning

heuristic search can benefit from representing uncertainty

scalar heuristic → belief distribution that represents uncertainty

# What is Real-time Heuristic Search?

Introduction

■ About Me

■ Real-time Search

Nancy

Conclusions

An example: path finding

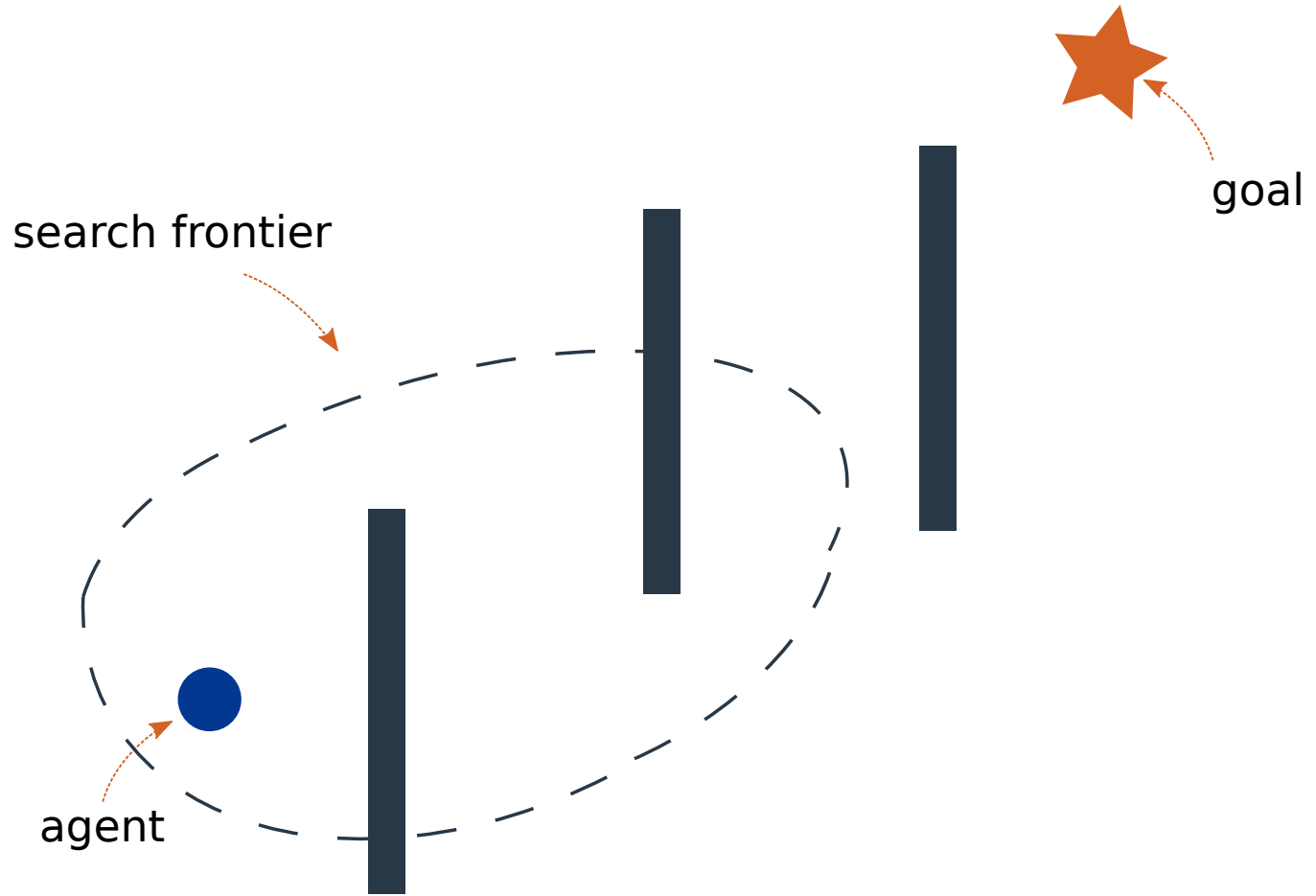


agent performs search for a bounded time

# What is Real-time Heuristic Search?

- Introduction
- About Me
- Real-time Search
- Nancy
- Conclusions

An example: path finding



# What is Real-time Heuristic Search?

Introduction

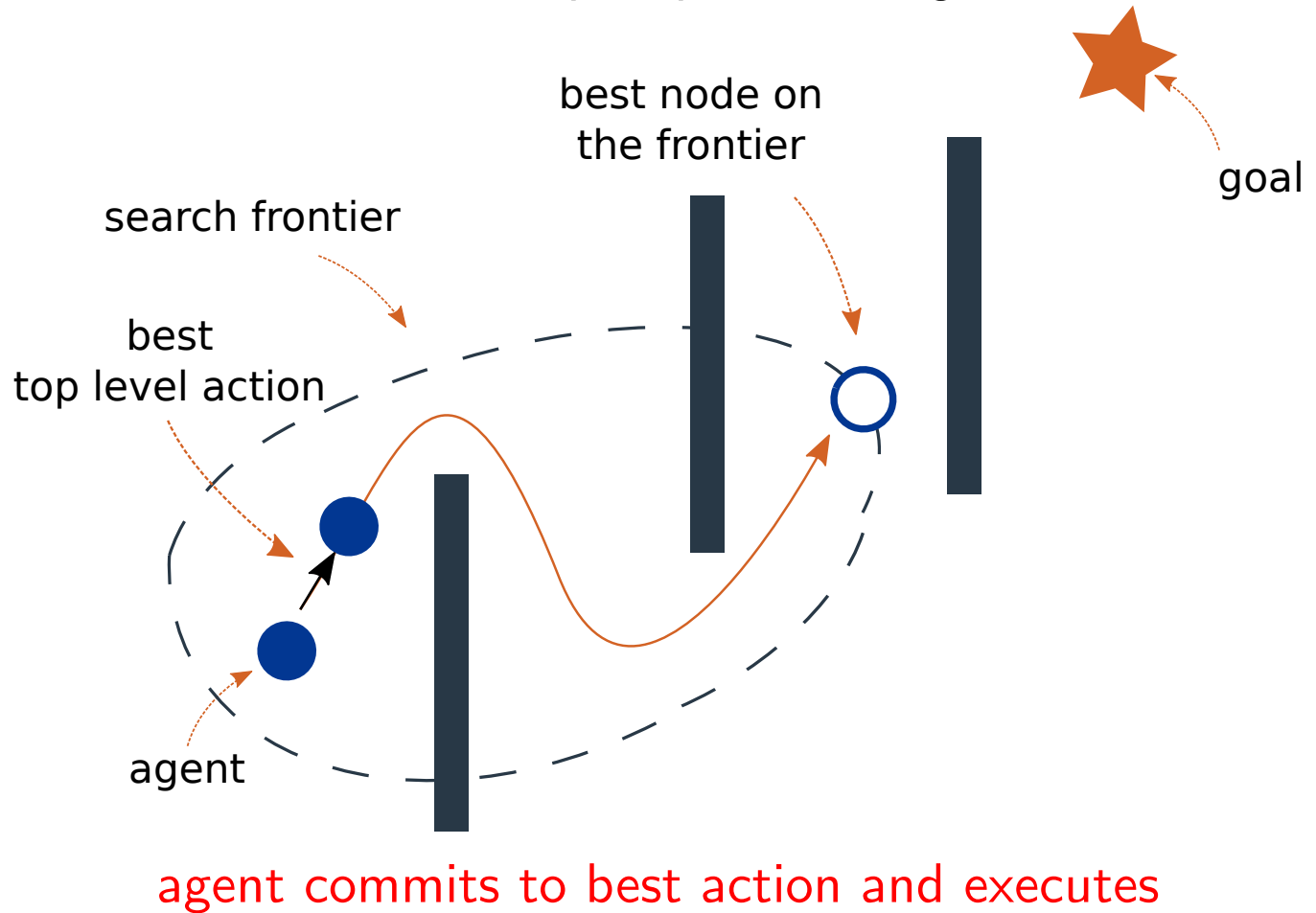
■ About Me

■ Real-time Search

Nancy

Conclusions

## An example: path finding



# What is Real-time Heuristic Search?

Introduction

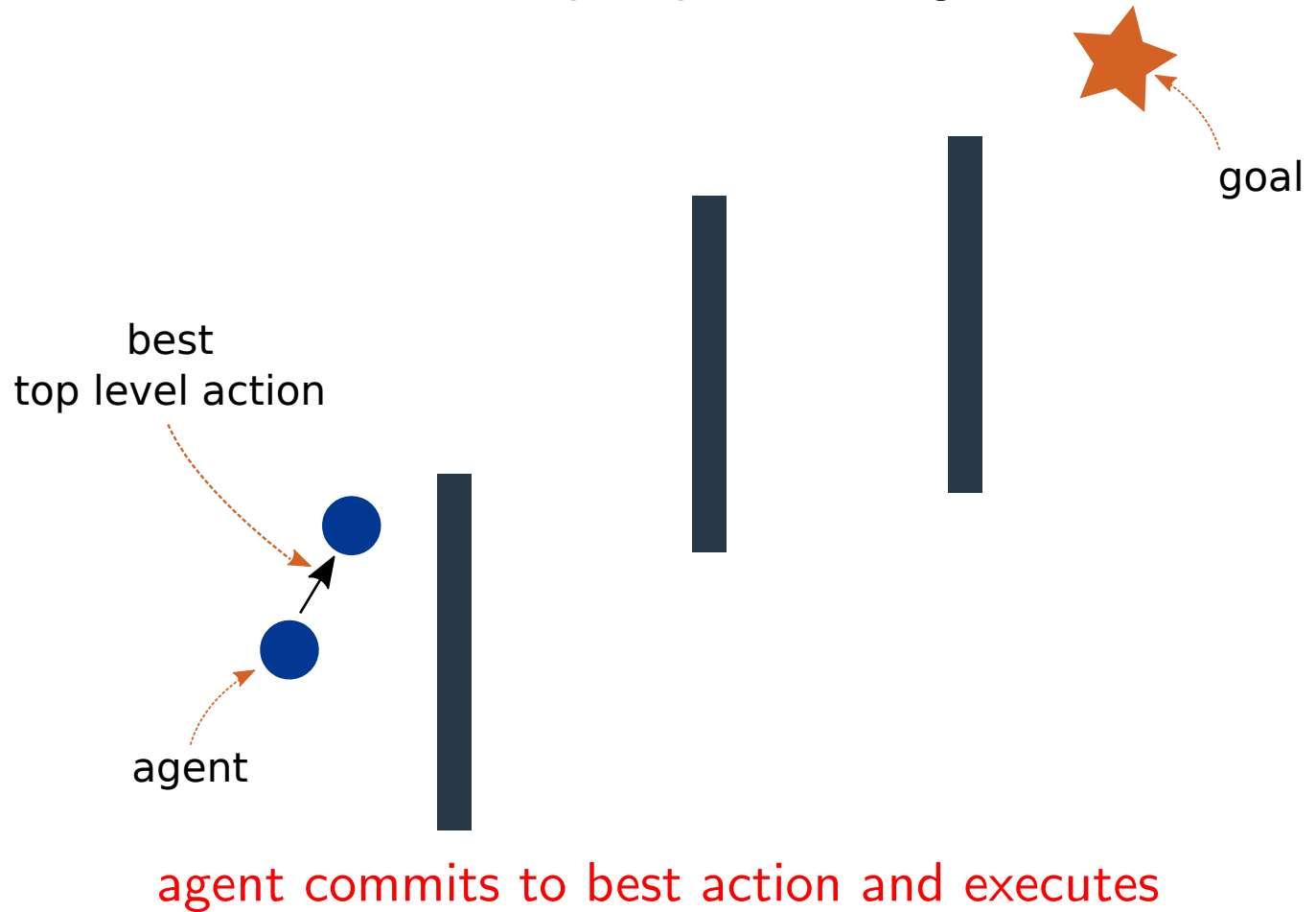
■ About Me

■ Real-time Search

Nancy

Conclusions

An example: path finding



# What is Real-time Heuristic Search?

Introduction

■ About Me

■ Real-time Search

Nancy

Conclusions

An example: path finding



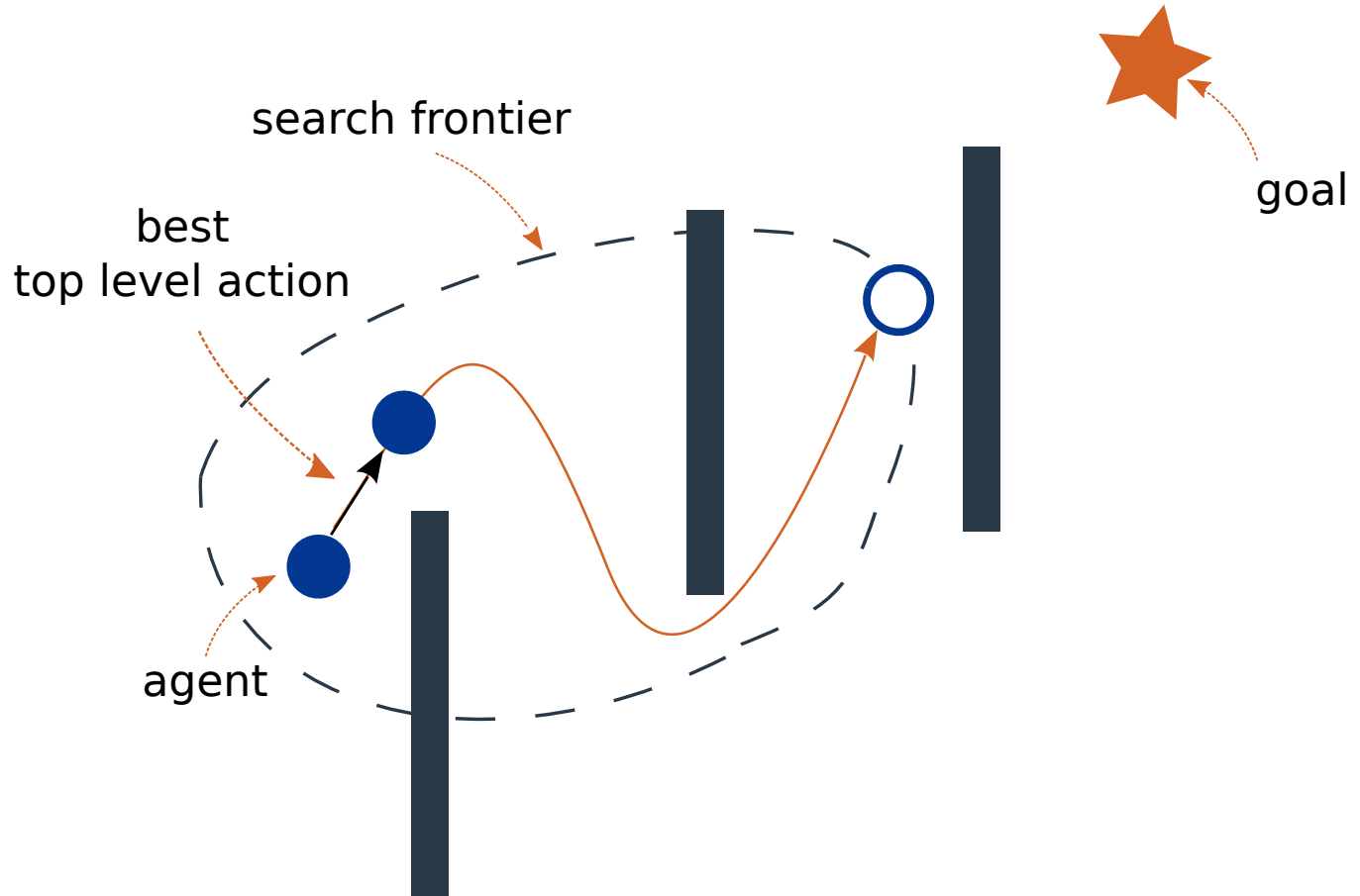
agent commits to best action and executes



# What is Real-time Heuristic Search?

- Introduction
- About Me
- Real-time Search
- Nancy
- Conclusions

An example: path finding



online planning: interleaving search and action execution  
“receding horizon control”

# Motivation for Real-time Heuristic Search

---

Introduction

■ About Me

■ Real-time Search

Nancy

Conclusions

## Real-time heuristic search:

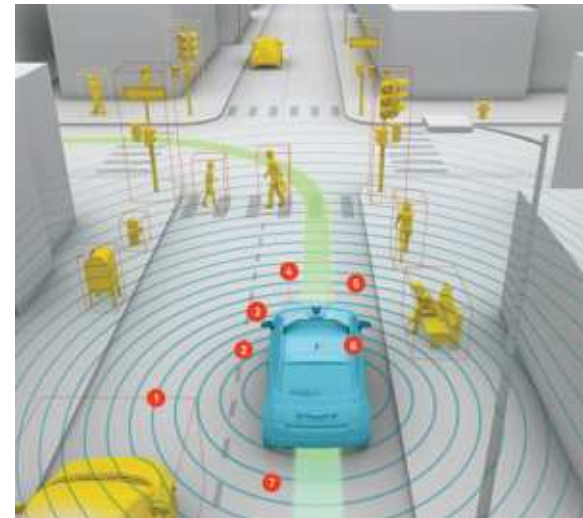
return the next action within a **time bound**

## Applications:

interacting with humans

dynamic environment

- autonomous vehicle
- inaccurate sensor
- update model online



Introduction

**Nancy**

- Decision-making
- Lookahead
- The Beliefs
- Results

Conclusions

# Real-time Search as Decision-making Under Uncertainty: The Nancy Framework

# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

---

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

three phases:

1. Lookahead Phase:  
expands nodes with minimum  $f$   
to explore the search space

# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

---

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

three phases:

1. Lookahead Phase:

expands nodes with minimum  $f$   
to explore the search space

2. Decision-making Phase:

backup the minimum  $f$  from search frontier ('minimin')  
select top level action with minimum  $f$  to execute

# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

three phases:

1. Lookahead Phase:  
expands nodes with minimum  $f$   
to explore the search space
2. Decision-making Phase:  
backup the minimum  $f$  from search frontier ('minimin')  
select top level action with minimum  $f$  to execute
3. Learning Phase:  
update heuristic values  
(to escape local minima and avoid infinite loops)

# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

three phases:

1. Lookahead Phase:

expands nodes with minimum  $f$   
to explore the search space

2. Decision-making Phase:

backup the minimum  $f$  from search frontier ('minimin')  
select top level action with minimum  $f$  to execute

3. Learning Phase:

update heuristic values  
(to escape local minima and avoid infinite loops)

repeat until at a goal

# A Classic Approach: LSS-LRTA\* (Koenig&Sun 2008)

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

three phases:

1. Lookahead Phase:  
expands nodes with minimum  $f$   
to explore the search space
2. Decision-making Phase:  
backup the minimum  $f$  from search frontier ('minimin')  
select top level action with minimum  $f$  to execute
3. Learning Phase:  
update heuristic values  
(to escape local minima and avoid infinite loops)

repeat until at a goal

derived from offline search, but optimal for online?



# Decision-making Phase: A Troublesome Example

Introduction

Nancy

Decision-making

Lookahead

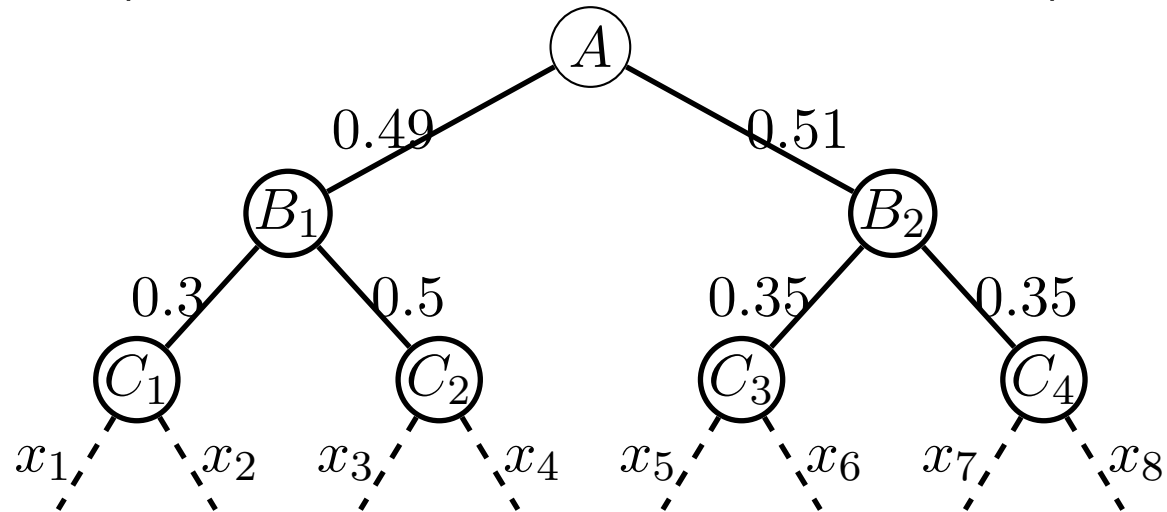
The Beliefs

Results

Conclusions

Should an agent at  $A$  move to  $B_1$  or  $B_2$ ?

( $x_i$  are unknown but i.i.d. uniform 0-1)



random tree domain (Pemberton & Korf 1995)

$f = g + h = g + 0$  is lower bound on optimal plan cost

# Decision-making Phase: A Troublesome Example

Introduction

Nancy

Decision-making

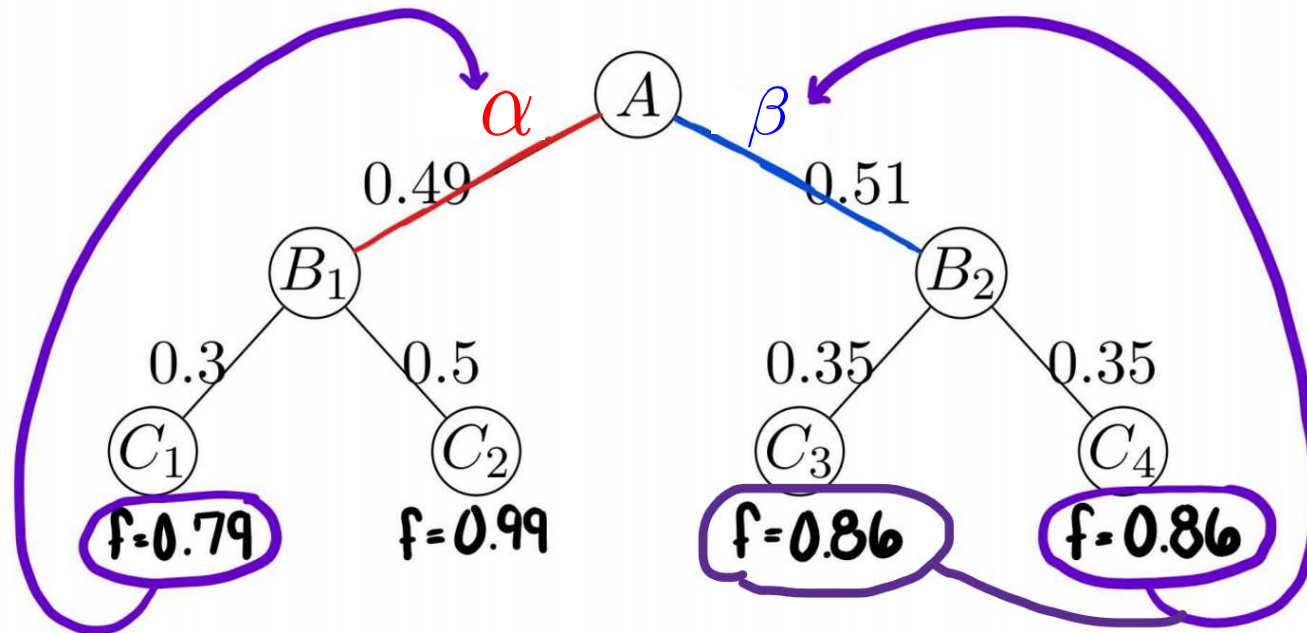
Lookahead

The Beliefs

Results

Conclusions

Should an agent at  $A$  move to  $B_1$  or  $B_2$ ?  
( $x_i$  are unknown but i.i.d. uniform 0-1)



decision theory says minimize expected value

lower bound: **not suitable for rational action selection**

# Decision-making Phase: A Troublesome Example

Introduction

Nancy

Decision-making

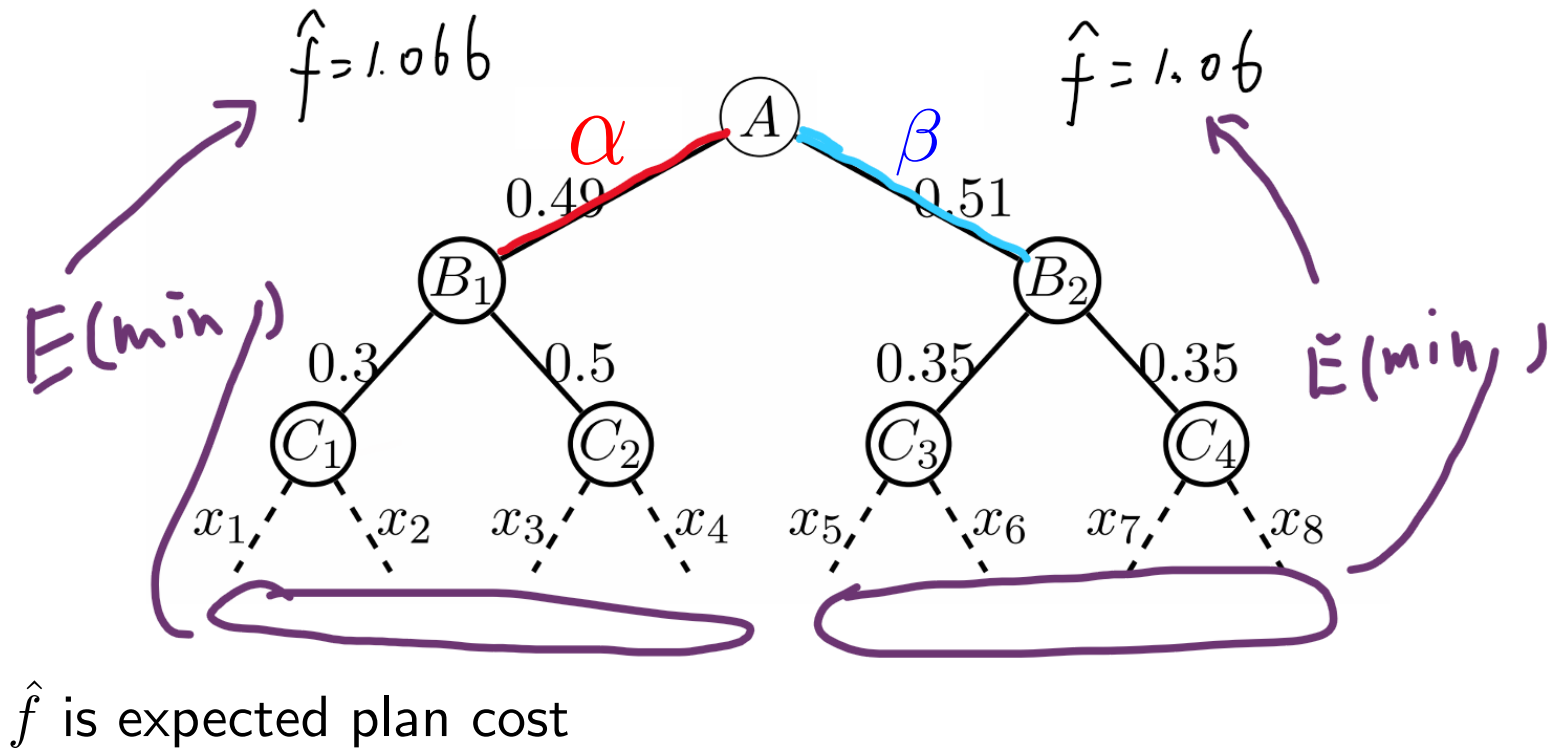
Lookahead

The Beliefs

Results

Conclusions

Should an agent at  $A$  move to  $B_1$  or  $B_2$ ?  
( $x_i$  are unknown but i.i.d. uniform 0-1)



# Decision-making Phase: A Troublesome Example

Introduction

Nancy

Decision-making

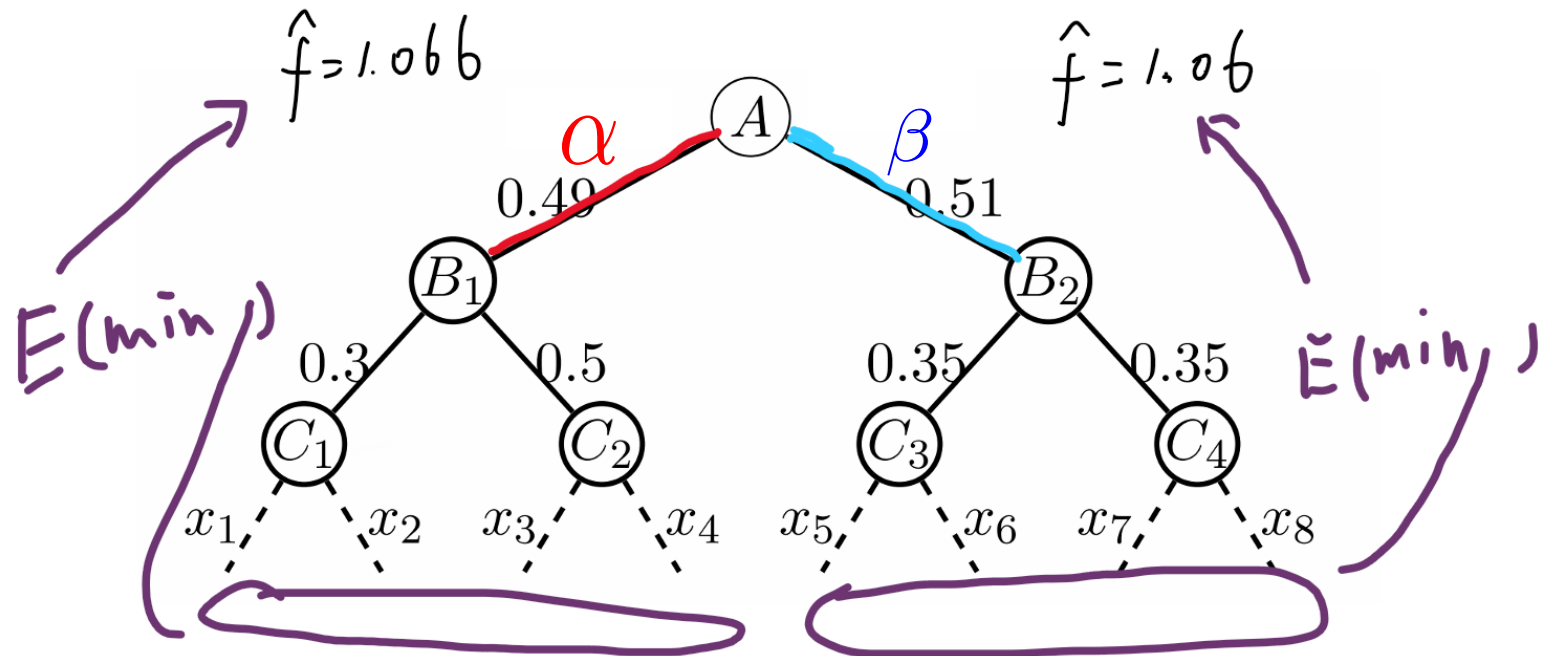
Lookahead

The Beliefs

Results

Conclusions

Should an agent at  $A$  move to  $B_1$  or  $B_2$ ?  
( $x_i$  are unknown but i.i.d. uniform 0-1)



$\hat{f}$  is expected plan cost

$f$  is not the answer: should minimize expected value!

plan under time pressure  $\rightarrow$  bounded rationality

# Lookahead Phase: A Troublesome Example

Introduction

Nancy

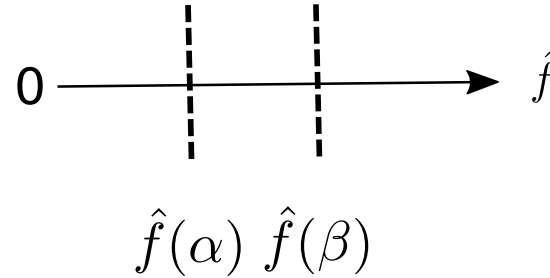
■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions



$\hat{f}$  is expected value

Should an agent expand nodes under  $\alpha$  or  $\beta$ ?

# Lookahead Phase: A Troublesome Example

Introduction

Nancy

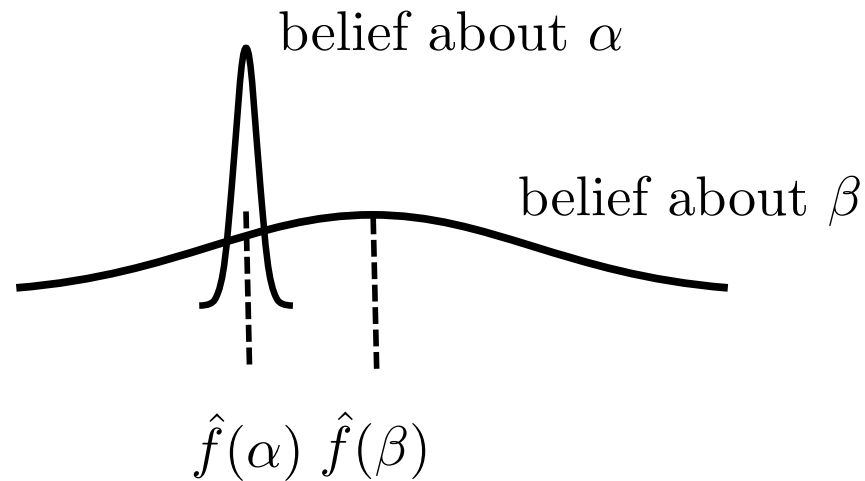
■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions



$\hat{f}$  is expected value

Should an agent expand nodes under  $\alpha$  or  $\beta$ ?

$\hat{f}$  is not the answer: what to do?  
want to maximize value of information  
need to consider uncertainty of estimates

## Risk-based lookahead <sup>1</sup>:

want to maximize value of information

expand nodes which minimize expected regret

relies on belief of values

choose expansions that decrease uncertainty in beliefs

---

<sup>1</sup> Real-time Planning as Decision-making Under Uncertainty, Andrew Mitchell, Wheeler Ruml, Fabian Spaniol, Joerg Hoffmann, and Marek Petrik, AAI, 2019.

# Backup Rules: Nancy

Introduction

Nancy

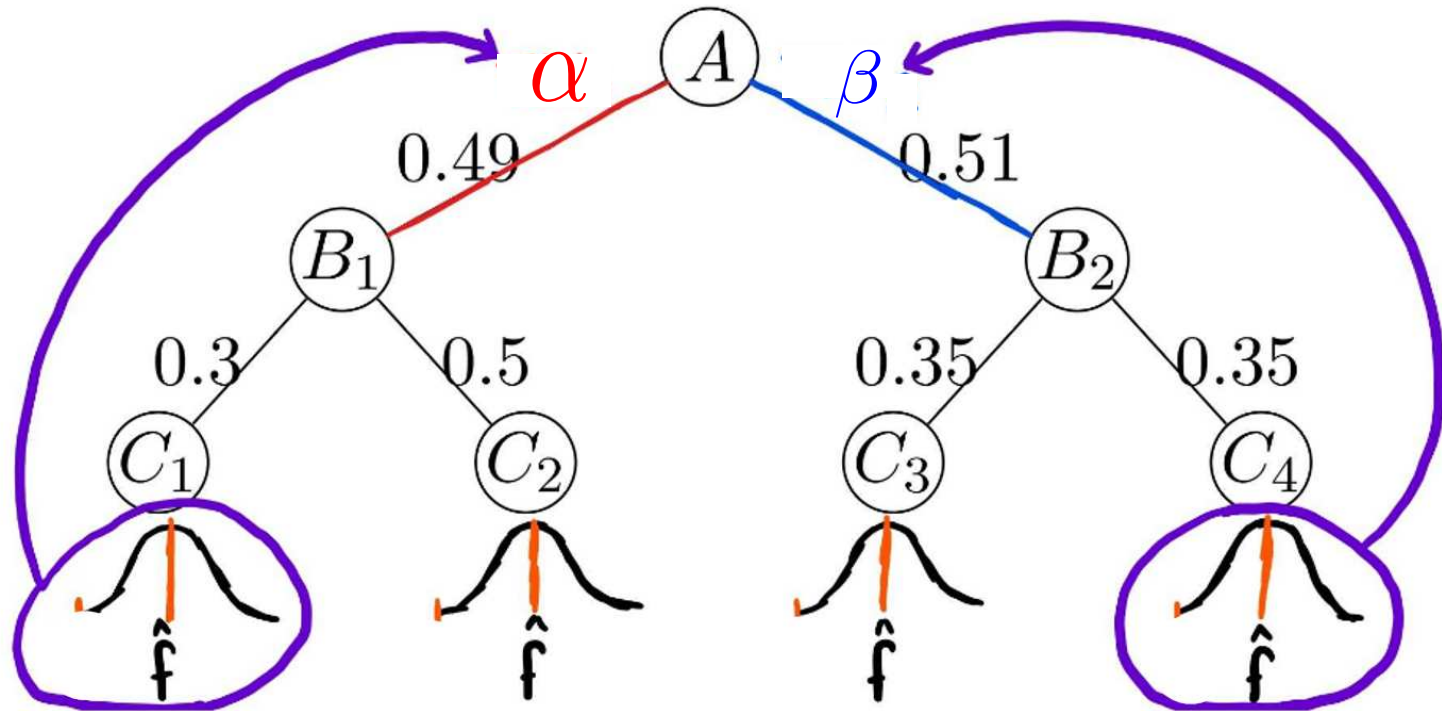
■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions



**Nancy:**

parent  $\leftarrow$  belief with minimum  $\hat{f}$  among successors

conveys an entire belief distribution



# How to Form The Belief Distribution?

---

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

Heuristic values: scalar  $\rightarrow$  probability distribution (belief)

But where do beliefs come from?

# How to Form The Belief Distribution?

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

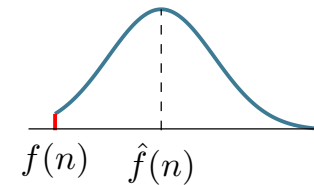
Conclusions

Heuristic values: scalar  $\rightarrow$  probability distribution (belief)

But where do beliefs come from?

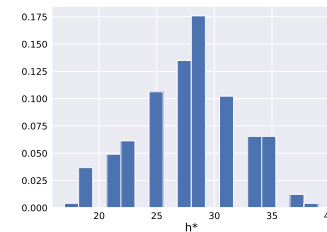
Nancy:

truncated Gaussian based on  $\hat{f}$  and  $f$ ,  
few parameters allows **online learning**



Data-Driven Nancy<sup>2</sup>:

expressive histogram,  
many parameters requires **offline learning**



<sup>2</sup>Beliefs We Can Believe In: Replacing Assumptions with Data in Real-Time Search, Maximilian Fickert, Tianyi Gu, Leonhard Staut, Wheeler Ruml, Joerg Hoffmann, and Marek Petrik, AAAI, 2020.

# Mean Solution Cost on Planning Domains

Introduction

Nancy

■ Decision-making

■ Lookahead

■ The Beliefs

■ Results

Conclusions

Domain	Lookahead	LSS-LRTA*	Nancy	Nancy (DD)
Blocksw.	100	46	<b>33</b>	38
	300	36	<b>30</b>	34
	1000	30	32	<b>27</b>
Transport	100	631	615	<b>496</b>
	300	519	559	<b>485</b>
	1000	499	567	<b>422</b>
Transport (unit-cost)	100	48	40	<b>31</b>
	300	47	<b>30</b>	34
	1000	35	29	<b>27</b>
Elevators (unit-cost)	100	50	<b>35</b>	39
	300	32	<b>29</b>	30
	1000	34	27	<b>26</b>

Both version of Nancy outperform conventional approach!

Introduction

Nancy

**Conclusions**

■ Summary

# Conclusions

Examples of **using distribution to guide search**:

- real-time planning: Nancy, DDNancy
- suboptimal search: XES (IJCAI-21 paper 994)
- robotics: BEAST (IROS-17)

Examples of **using distribution to guide search**:

- real-time planning: Nancy, DDNancy
- suboptimal search: XES (IJCAI-21 paper 994)
- robotics: BEAST (IROS-17)

Exciting time in AI!

- Planning, RL, ML, Robotics

Examples of **using distribution to guide search**:

- real-time planning: Nancy, DDNancy
- suboptimal search: XES (IJCAI-21 paper 994)
- robotics: BEAST (IROS-17)

Exciting time in AI!

- Planning, RL, ML, Robotics

Much work needs to be done!

- data-driven + planning
- statistics + model-based approach

# Questions?

---

Introduction

Nancy

Conclusions

Questions

■ Questions?





Introduction

Nancy

Conclusions

**Back-up Slides**

■ Completeness

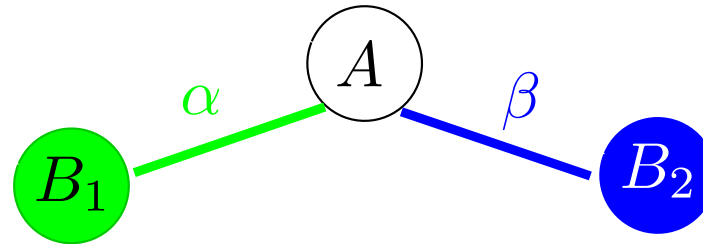
■ Search

# Back-up Slides

# Risk-based Lookahead Example

---

expand under  $\alpha$  or  $\beta$ ?



[Introduction](#)

[Nancy](#)

[Conclusions](#)

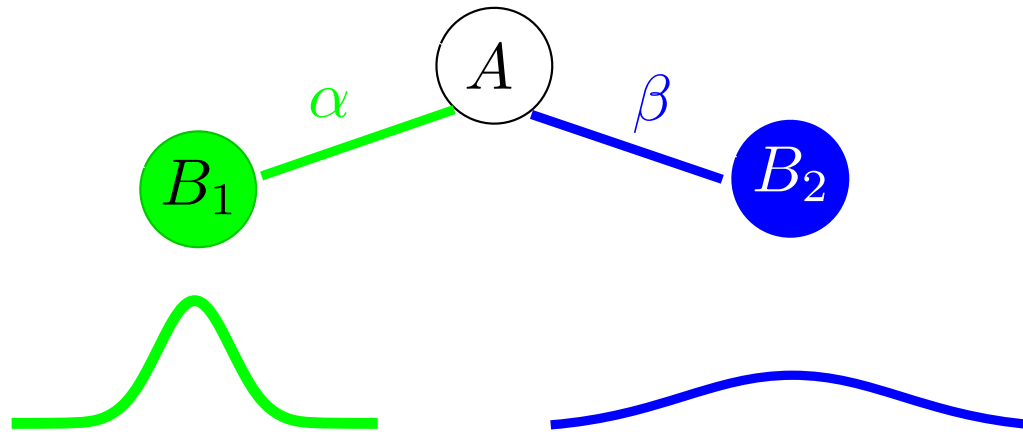
[Back-up Slides](#)

■ [Completeness](#)

■ [Search](#)

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

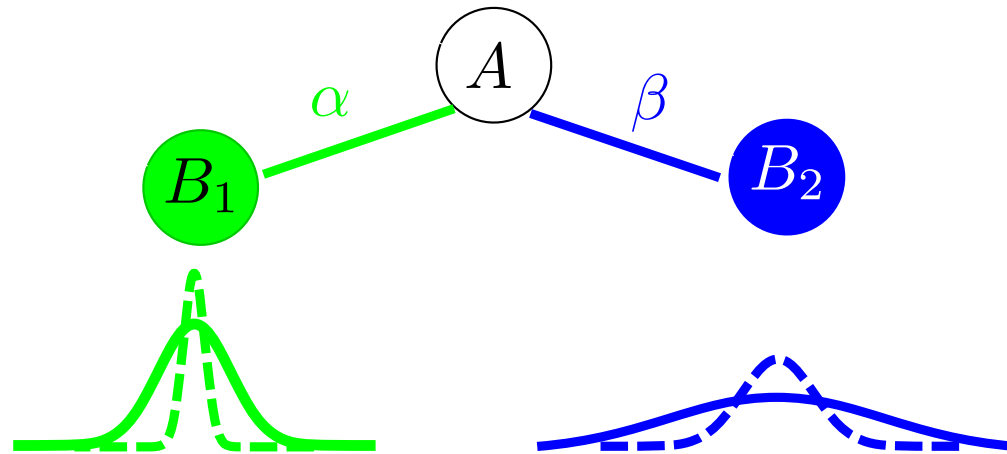


need 2 things:

- 1) current beliefs
- 2) estimate of how beliefs might change with search

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

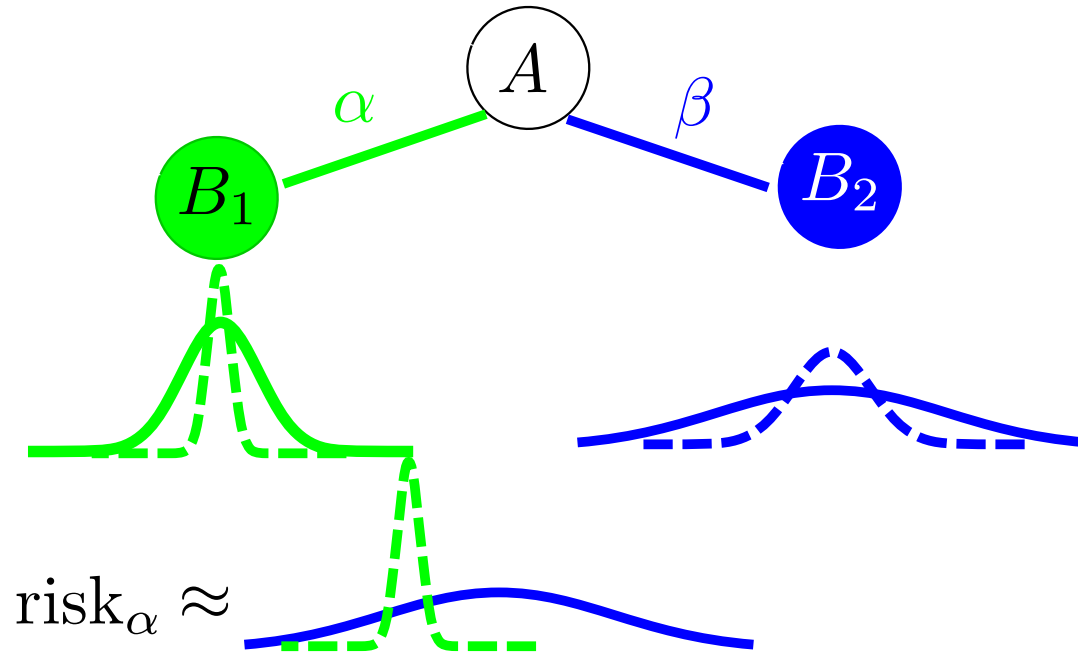


need 2 things:

- 1) current beliefs
- 2) estimate of how beliefs might change with search

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

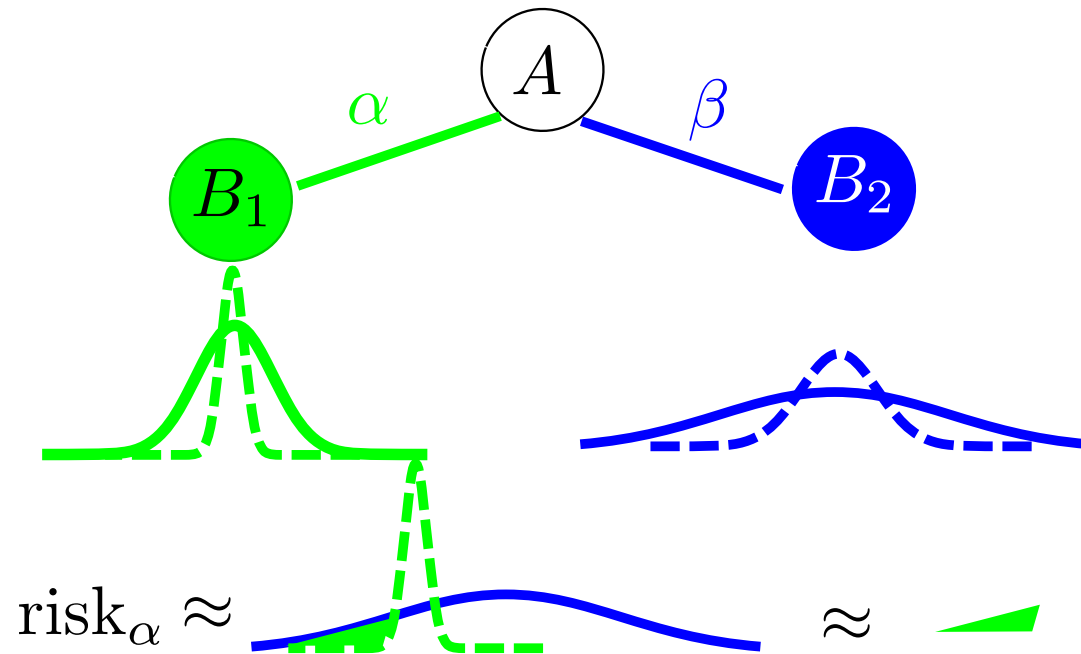


**Risk:** expected regret if a suboptimal action is selected  
 $\alpha$  is TLA with lowest expected value, other is  $\beta$

$$\mathbb{E} \left[ \underbrace{f^*(\alpha) - f^*(\beta)}_{\text{what is our regret}} \mid \underbrace{f^*(\beta) < f^*(\alpha)}_{\text{in cases when } \alpha \text{ not best}} \right]$$

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?

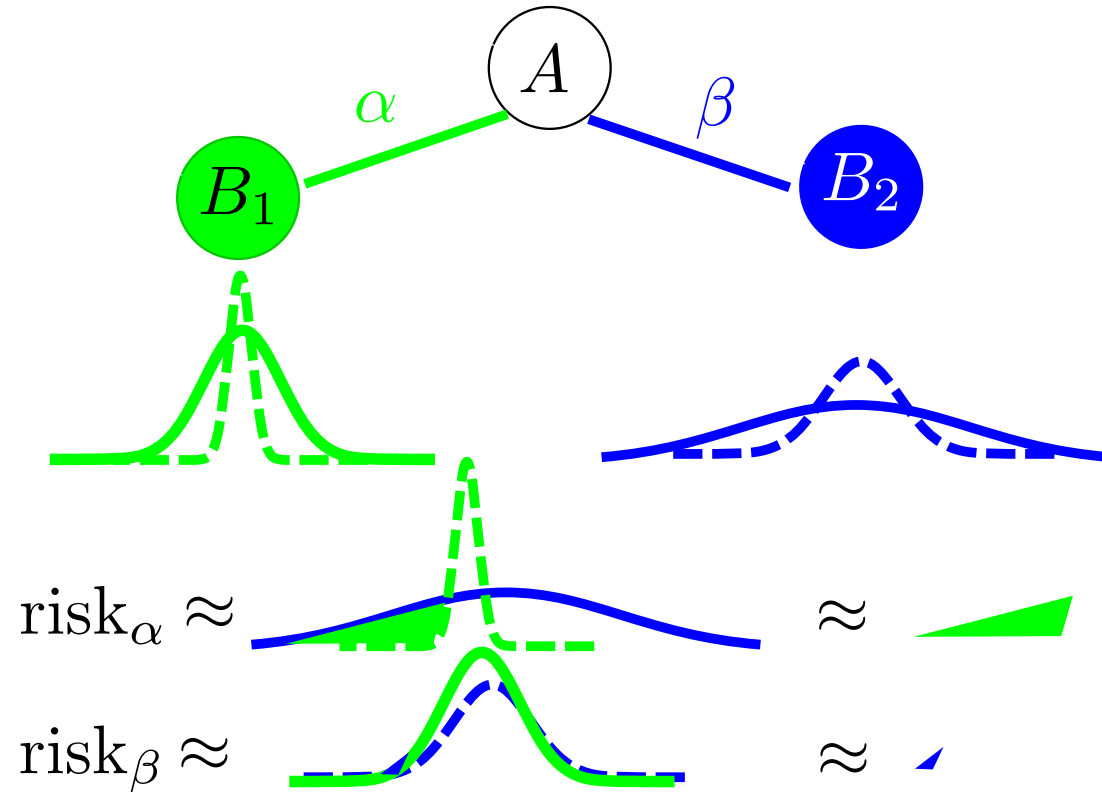


**Risk:** expected regret if a suboptimal action is selected  
 $\alpha$  is TLA with lowest expected value, other is  $\beta$

$$\mathbb{E} \left[ \underbrace{f^*(\alpha) - f^*(\beta)}_{\text{what is our regret}} \mid \underbrace{f^*(\beta) < f^*(\alpha)}_{\text{in cases when } \alpha \text{ not best}} \right]$$

# Risk-based Lookahead Example

expand under  $\alpha$  or  $\beta$ ?



expand under the TLA that minimizes risk!  
expand under  $\beta$ !

Introduction

Nancy

Conclusions

Back-up Slides

■ Completeness

■ Search

# Completeness proof

---

Introduction

Nancy

Conclusions

Back-up Slides

■ Completeness

■ Search

**Lemma 1** *Under assumptions of goal-awareness and finite state space, if a real-time search algorithm is incomplete, it must have a circulating set  $S_o$ .*

**Lemma 5** *Under our assumptions, a reasonable real-time search algorithm cannot have a circulating set.*

**Theorem 1** *Under our assumptions, a reasonable real-time search algorithm will eventually reach a goal.*

**Lemma 7** *Nancy is a reasonable real-time search algorithm.*

**Lemma 8** *LSS-LRTA\* is a reasonable real-time search algorithm.*

**This proof applies to any LSS-LRTA\*-style algorithm: no longer need heuristic consistency!**



# Search Domains

---

[Introduction](#)

[Nancy](#)

[Conclusions](#)

[Back-up Slides](#)

Completeness

Search

sliding tile puzzle

uniform, heavy ,inverse

pancake puzzle

different size

racetrack

reminiscent of autonomous driving

# Comparison to IE and MCTS on Classic Search Domains

Introduction

Nancy

Conclusions

Back-up Slides

■ Completeness

■ Search

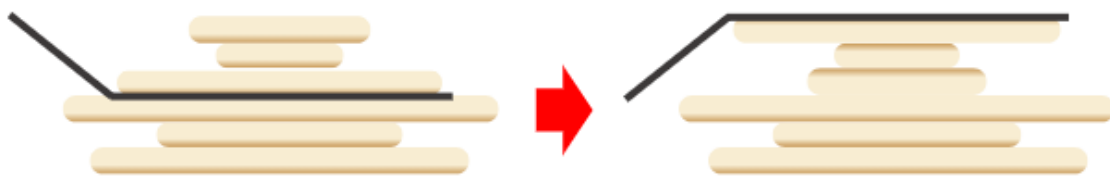
pancake puzzle



Start



Goal



Allowed Move

# Comparison to IE and MCTS on Classic Search Domains

Introduction

Nancy

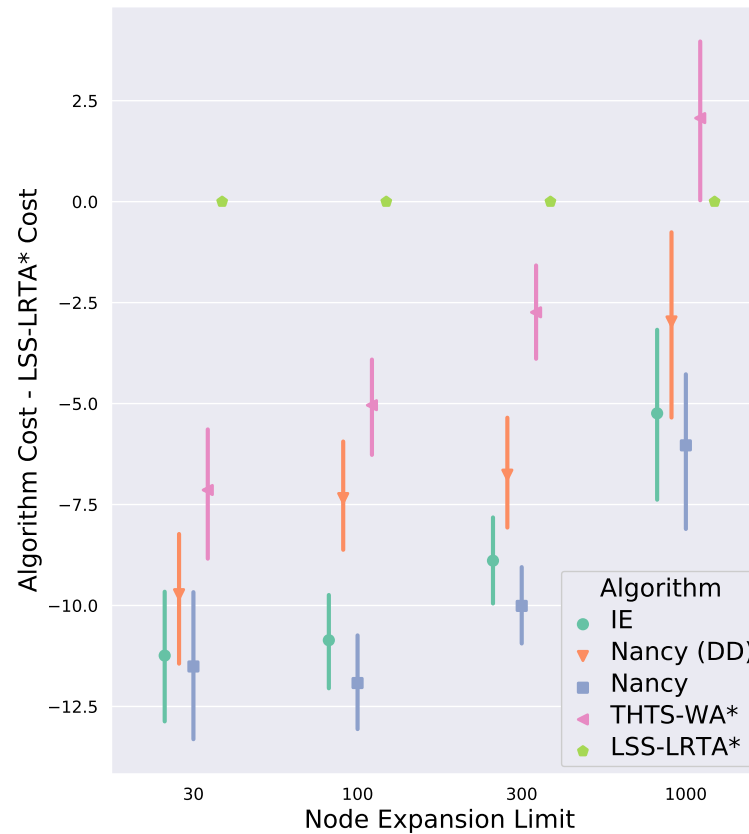
Conclusions

Back-up Slides

■ Completeness

■ Search

## 40 Pancake



Nancy outperforms conventional approaches and MCTS<sup>3</sup>

<sup>3</sup>Real-time Planning as Data-driven Decision-making, Maximilian Fickert, Tianyi Gu, Leonhard Staut, Sai Lekyang, Wheeler Ruml, Joerg Hoffmann, and Marek Petrik, Bridging the Gap Between AI Planning and Reinforcement Learning (PRL), 2020.