CS 758/858: Algorithms

`http://www.cs.unh.edu/~ruml/cs758`

Games

games in 'extensive form'

Equilibria

Each agent gives its best response
Nash equilibrium: no agent has incentive to deviate. Under best response dynamics, stable.
in K+Tfig 12.8b, two NE of different values, only one is a "social optimum"
In K+Tfig12.10, single N.E is H(k) = Theta(log k) worse than social optimum (which is not an equilibrium). Theorem that this is worst possible.
Price of stability/anarchy: best NE / social optimum
NE don't always exist.
Not known whether one can find NE in poly time, even a bad one
finding a Nash equilibrium is PPAD-complete (in subset of NP but not NP-complete)
if one seeks a Nash equilibrium that maximizes the sum of player utilities, or one that uses a given strategy with positive probability, then the problem becomes NP-complete

Braess's Paradox

adding a road to a network can slow traffic!
ie, N.E. is not social optimum

[this has been observed in practice: closing roads can improve flow]

Stable Matching

Stable Matching

Given preferences of every man over every woman and of every woman over every man, pair up everyone
perfect matching = each exactly once

"self-enforcing" solutions: despite parties acting in their own self-interest
Stable: there is no man m and woman w for which m prefers w to his match and w prefers m to her match

National Resident Matching Program, 1950s

The Gale-Shapley Algorithm (1962)

1. all m and w are free
2. while there exists a free m who hasn't proposed to every w
3. choose such a m
4. w is m's highest ranked to whom he has not yet proposed
5. if w is free, (m, w) are engaged
6. else (w engaged to some m')
7. if w prefers m to m', m' free and (m, w) engaged
8. else m remains free

G-S Properties

Property 1: w free, then engaged to higher and higher ranked m
Property 2: women to whom m proposes are worse and worse ranked
Theorem 1: G-S terminates in $O(n^2)$ time
Proof: each iteration is a new proposal (m has never proposed to w before). There are at most n^2 possible proposals.
Lemma 1: if a man m is free, there is a woman to whom he has not proposed.
Proof: if m has proposed to every woman but is not engaged, all women must already be engaged. But this cannot be as $\#$ men $= \#$ women
Lemma 2: G-S returns a perfect matching
Proof: always a matching. None double-assigned as we only assign to free men and women dump worse guy. No loners as can only terminate when every man has proposed to every woman and lemma 1 implies such a man cannot be free

G-S Properties

Theorem 2: G-S computes stable matching

Proof: Assume of (m, w) and (m_1, w_1) that m prefers w_1 and w_1 prefers m. Note m's last proposal was to w. But note that m must have previously proposed to w_1 and been rejected in favor of some m_2. Either $m_2 = m_1$, which would mean w_1 ranks m_1 higher than m (contradiction), or m_2 is ranked higher than m and m_1 is ranked higher than m_2 (property 1), which means that w prefers m_1 to m (contradiction). Thus there cannot be instability.

Corollary: a stable matching always exists. (Sometimes many)

Property 3: Unfair for women: if men each most prefer different women, women's preferences ignored! Side that proposes gets best possible matches, other gets worst. This is regardless of how free man is chosen.

Property 4: The set of stable matchings corresponds to the core of a cooperative game, where no subset of agents can deviate and form a coalition that would make them all better off.

Mechanism Design

Auctions

negotiation with:
bidding rules: how offers are made
clearing rules: when do trades occur and what are they
information rules: who knows what when

efficient = allocate goods to those who value them the most
optimal auction = revenue maximizing

English auction: agents bid up until end, buyer = highest and pays his bid. "Open outcry"
Japanese: price rises, buyers drop out, ends when only one buyer is left, they pay that price
Dutch auction: price lowers until someone buys, first bid gets it
first-price sealed bid: submit to auctioneer, highest wins
second-price = **Vickrey:** highest wins, pays second highest
Vickrey–Clarke–Groves: generalization to multiple items

Auction Properties

dominant-strategy incentive-compatibility (DSIC): best to tell truth regardless of what others do
Bayes-Nash incentive-compatibility (BNIC): there is an equilibrium in which everyone tells truth.
second-price auction = eBay if all agents use bidding proxies
¡¿ Japanese: truthful with adversary who bids your valuation
dutch and English auctions strategically equivalent
simple majority vote between two choices is DSIC
second-price auction is DSIC, first-price isn't

Theorem: in a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly from [0,1], (0.5 v_1, 0.5 v_2) is a Bayes-Nash equilibrium strategy profile (eg, each bidder bids half their valuation). Note that this is not truthful!
Proof: see book.

Vickrey Auctions Are Incentive Compatible

Theorem: in second price auction with independent private valuations, truth telling is a dominant strategy
Proof: Assume others bid arbitrarily. Consider i's response. Case 1: i's valuation is higher than the highest other. No advantage to bidding higher, because already winning and doesn't change price paid. Bidding lower either a) doesn't change price paid if still win or b) results in losing and paying 0 (negative utility compared to winning at valuation). So i cannot gain and might lose by bidding dishonestly. Case 2: i's valuation is less than another's bid. If i bid less, would still lose. If he bid more, willl either not change outcome or could win and pay more than valuation (negative utility). So i cannot gain and might lose by bidding dishonestly in this case too.

Note that proof does not depend on the risk attitudes of the agents.

Social Choice Theory

Gibbard–Satterthwaite theorem: Any deterministic ordinal electoral system that chooses a single winner has at least one of the following properties:
1. The rule is dictatorial, i.e. there exists a distinguished voter who can choose the winner; or
2. The rule limits the possible outcomes to two alternatives; or
3. The rule is not straightforward, i.e. there is no single always-best strategy (that does not depend on other voters' preferences or behavior).

Arrow's impossibility theorem: If there are at least three alternatives, no ranking-based decision rule can satisfy:
1. Pareto-efficiency
2. Non-dictatorship
3. Independence of irrelevant alternatives

Nope!