More About DFAs and Regular Languages

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Theorem   The language recognized by a DFA
is a regular language.

Proof   by deconstructing DFA to
produce regular expressions

1. add new initial and final states
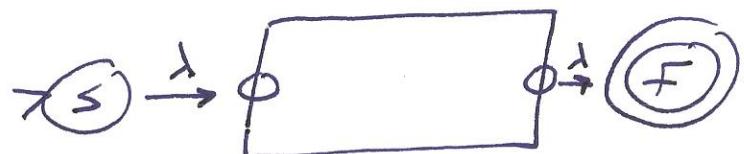
actually could be
multiple final
states in the
original DFA

2.

$$i \xrightarrow{x} j \quad \Longrightarrow \quad i \xrightarrow{x/y} j$$

$$ \xrightarrow{y} $$
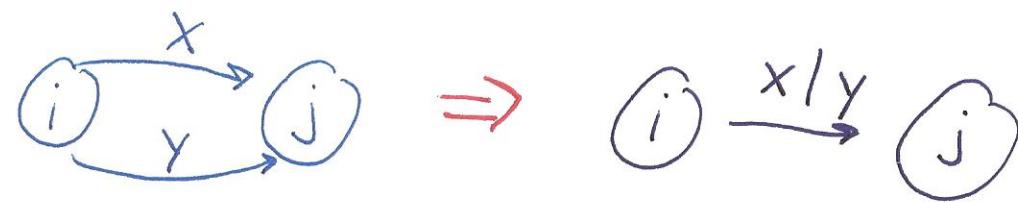
3. now eliminating original states,
   one at a time

$$ \overset{y}{(i) \xrightarrow{x} (j) \xrightarrow{z} (k)} \;\Rightarrow\; (i) \xrightarrow{(xy^*z)|w} (k) $$

with w labeling the arc from i to k.
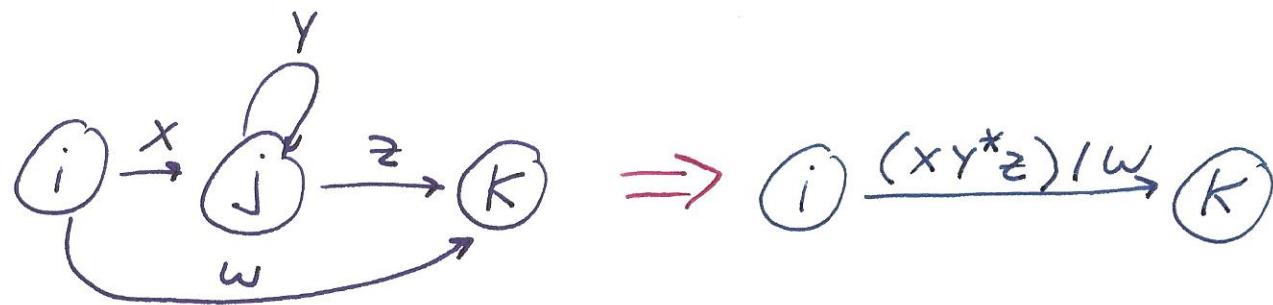
when removing a state, must consider
   all pairs of states connected via
   the state being removed.

4. if state s is connected to F
   then label on s →f is the
   regular expression recognized
   by the DFA.

   if state s is not connected to f
   then the DFA recognizes ϕ.

exemple

Diagram 1 (top):

$S \xrightarrow{\quad ab^*a \quad} \bigcirc \xrightarrow{\ b\ } \bigcirc \xrightarrow{\ \lambda\ } (F)$

self-loop on middle state: $(bb^*a)|a$

Diagram 2 (middle):

$S \xrightarrow{\quad (ab^*a)((bb^*a)|a)^* b \quad} \bigcirc \xrightarrow{\ \lambda\ } (F)$

Diagram 3 (bottom):

$S \xrightarrow{\quad (ab^*a)((bb^*a)|a)^* b \quad} (F)$

6