Floating-Point Addition

CS520

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$$\begin{array}{r} 2.34 \times 10^{2} \\ + \; 2.56 \times 10^{0} \\ \hline \end{array}$$

$$\begin{array}{r} 2.34\boxed{00} \times 10^{2} \\ 0.0256 \times 10^{2} \\ \hline 2.36\boxed{56} \times 10^{2} \\ \hline \end{array}$$

$$\boxed{2.37 \times 10^{2}}$$

Floating - Point Addition

1. shift significand of the smaller number
   to the right until exponents agree

2. add the significands

3. normalize the sum & check for
   overflow/underflow

$\overline{10,0\cdots}$

→

$\overline{0.00001\cdots}$

←

4. round the sum
   ↳ might then need another normalization

```
  7F2A AAAA
+ 78F8 7878
-----------
```

$$0\,\underbrace{111\;111\overset{0}{\cancel{1}}\,\overset{1}{0}}\,010\;\;1010\;\;1010\;\;1010\;\;1010\;\;1010$$

$$0\,\underbrace{111\;1000\;1}\,111\;\;1000\;\;0111\;\;1000\;\;1111\;\;1000$$

$$0000\;1101$$

$$\Delta_{exp} = 13_{10}$$

Guard bits

```
                       1 1111        1
   1.010 1010 1010 1010 1010 1010 |  |
 + 0.000 0000 0000 0111 1100 0011 |11| 000 1111 1000
   ---------------------------------     \_________/
   1.010 1010 1011 0010 0110 1101          sticky bit = 1
                               +1
   ---------------------------------
   1.010 1010 1011 0010 0110 1110
   -
```

```
0111 1111 0010 1010 1011 0010 0110 1110
  7    F    2    A    B    2    6    E   ✓
```

limitations of floating-point numbers

remember: they are *approximate*!

so: $x \cancel{==} y$ is problematic

$$|x-y| < \varepsilon$$

also problematic

$$\begin{array}{rr} & 1.2345 \times 10^{3} \\ - & 1.2341 \times 10^{3} \\ \hline & 0.0004 \times 10^{3} \end{array}$$

$$\longleftarrow$$

$$4.\underline{0}\,\underline{0}\,\underline{0}\,\underline{0} \times 10^{-1}$$

?

really just garbage

and floating-point addition is not associative

$$-1.5 \times 10^{38} + (1.5 \times 10^{38} + 1.0) =$$

$$-1.5 \times 10^{38} + 1.5 \times 10^{38} =$$

$$0.0$$

$$(-1.5 \times 10^{38} + 1.5 \times 10^{38}) + 1.0 =$$

$$0.0 + 1.0 =$$

$$1.0$$

To learn more:

take a course in Numerical Analysis

e.g. Math 753

For one disaster story:

google "Ariane 5"