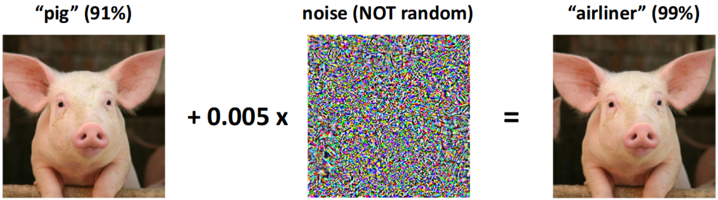
Robust Reinforcement Learning

Marek Petrik

Department of Computer Science
University of New Hampshire

DLRL Summer School 2019

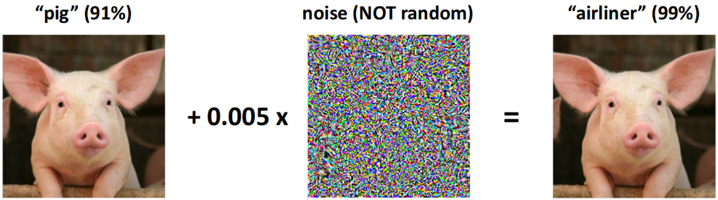
Adversarial Robustness in ML



[Kolter, Madry 2018]

Is this a problem?

Adversarial Robustness in ML



[Kolter, Madry 2018]

Is this a problem? Safety, security, trust

Are reinforcement learning methods robust?

Robustness

An algorithm is robust if it *performs well* even in the presence of *small errors* in inputs.

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Questions:

1. What does it mean to perform well?
2. What is a small error?
3. How to compute a robust solution?

Outline

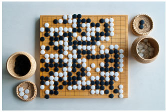
1. **Adversarial robustness in RL**
2. **Robust Markov Decision Processes**: How to solve them?
3. **Modeling input errors**: What is a small error?
4. **Other formulations**: What is the right objective?

Model-based approach to reliable off-policy sample-efficient tabular RL by learning models and confidence

Adversarial Robustness in RL

Robustness Not Important When . . .

- Control problems: inverted pendulum, . . .
- Computer games: Atari, Minecraft, . . .
- Board games: Chess, Go, . . .



Because

1. Mostly deterministic dynamics
2. Simulators are fast and precise:
   - Lots of data is available
   - Easy to test a policy
3. Failure to learn a good policy is cheap

Robustness Matters In Real World

1. Learning from logged data (batch RL):
   1.1 No simulator
   1.2 Never enough data
   1.3 How to test a policy? No cross-validation in RL
2. High cost of failure (bad policy)

Important in Real Applications

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Important in Real Applications

- Agriculture: Scheduling pesticide applications
- Maintenance: Optimizing infrastructure maintenance
- Healthcare: Better insulin management in diabetes
- Autonomous vehicles, robotics, ...

Example: Robust Pest Management

Agriculture: A challenging RL problem

1. Stochastic environment and delayed rewards
2. Must learn from data: No reliable, accurate simulator
3. One episode = one year
4. Crop failure is expensive

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Simulator: Using ecological population P models [Kery and Schaub, 2012]:

$$\frac{dP}{dt} = r\,P\left(1 - \frac{P}{K}\right)$$

Growth rate r, carrying capacity K, loosely based on spotted wing drosophila

Pest Control as MDP

States: Pest population: $[0, 50]$

Actions:

0 No pesticide

1-4 Pesticides P1, P2, P3, P4 with increasing effectiveness

Transition probabilities: Pest population dynamics

Reward:

1. Crop yield minus pest damage
2. Spraying cost: P4 more expensive than P1

MDP Objective: Discounted Infinite Horizon

Solution: Policy π maps *states* $\rightarrow$ *actions*
Objective: Discounted return:

$$\mathrm{return}(\pi) = \mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t \, \mathsf{reward}_t\right]$$

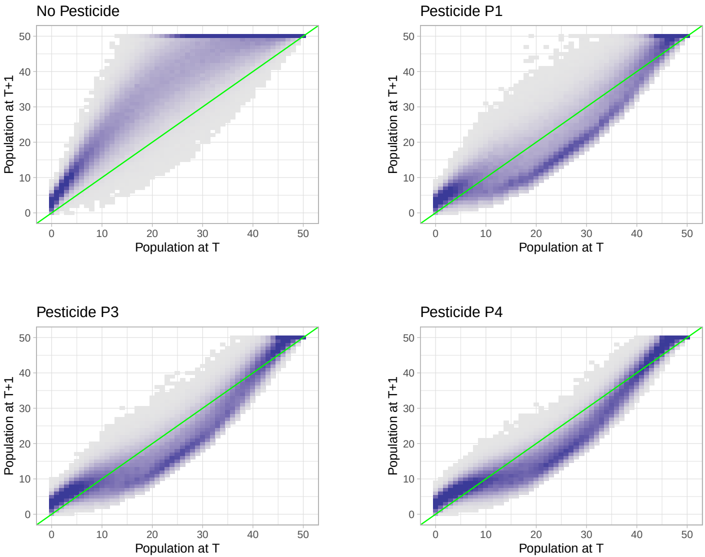
Optimal solution: Optimal policy

$$\pi^\star \in \arg\max_{\pi} \; \mathrm{return}(\pi)$$

Value function: v maps *states* $\rightarrow$ expected return
Bellman optimality:

$$v(s) = \max_a \left(r_{s,a} + \gamma \cdot p_{s,a}^{\mathsf{T}} v\right)$$

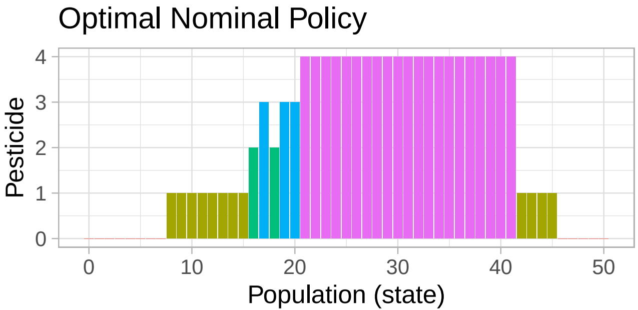
Transition Probabilities



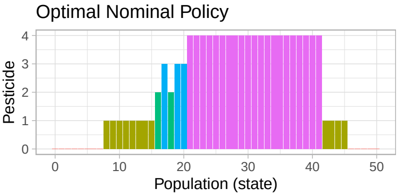
Computing Optimal Policy

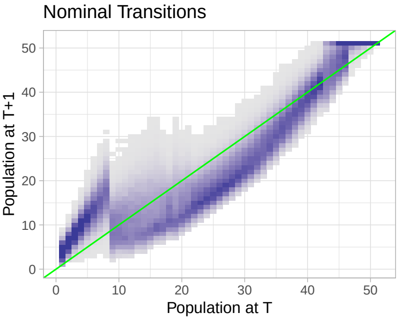
Algorithms: Value iteration, Policy iteration, Modified (optimistic) policy iteration, Linear programming

Return: $\mathbf{\$8,820}$

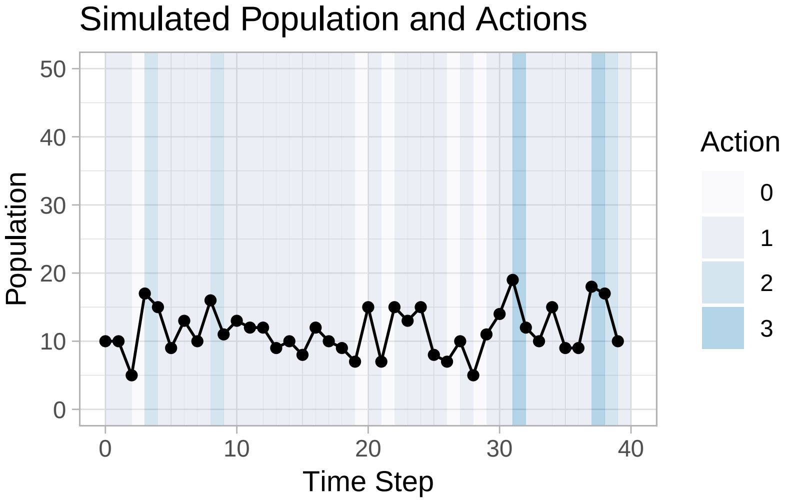


Optimal Management Policy





Simulated Optimal Policy



Is It Robust?

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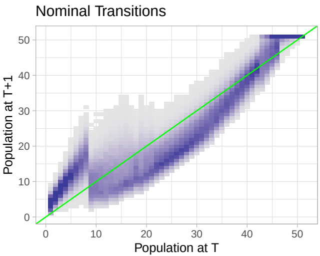

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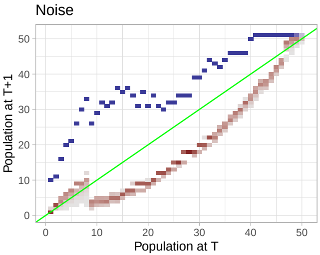
$L_1 \leq 0.05$



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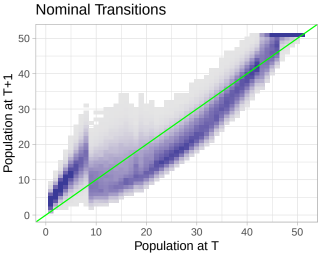

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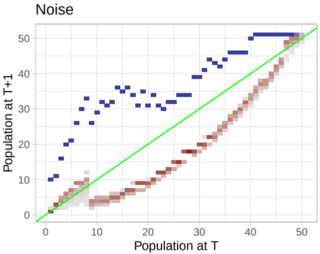
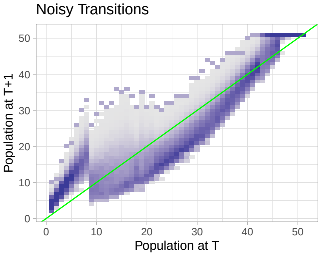


Return: $-\$6,725$



Adversarial Robustness for Reinforcement Learning

"An algorithm is robust if it performs well even in the presence of small errors in inputs. "

Robust optimization: Best π with respect to the inputs with all possible small errors:

$$\max_{\pi} \min_{P,r} \left\{ \mathrm{return}(\pi, P, r) \;:\; \begin{array}{c} \|\bar{P} - P\| \le \textsf{small} \\ \|\bar{r} - r\| \le \textsf{small} \end{array} \right\}$$

Adversarial nature chooses P, r

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Related to regularization e.g. [Xu et al., 2010], risk [Shapiro et al., 2014], and is opposite of exploration (MBIE/UCRL2) e.g. [Auer et al., 2010]

Robust Representation

Nominal values $\bar{P}$, $\bar{r}$

Errors in rewards: e.g. [Regan and Boutilier, 2009]

$$\max_{\pi} \min_{r} \left\{ \mathrm{return}(\pi, \bar{P}, r) \;:\; \|r - \bar{r}\| \le \psi \right\}$$

Errors in transitions: e.g. [Iyengar, 2005a]

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Budget of robustness ψ is the error size

Reward Function Errors

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Using MDP dual linear program: [Puterman, 2005]

$$\begin{aligned} \max_{u \in \mathbb{R}^{SA}} \quad & \min_{r \in \mathbb{R}^{SA}} \{ r^{\mathsf{T}} u \;:\; \|r - \bar{r}\| \le \psi \} \\ \text{s.t.} \quad & \sum_a (\mathbf{I} - \gamma P_a^{\mathsf{T}}) u_a = p_0 \\ & u \ge \mathbf{0} \end{aligned}$$

Reward Function Errors

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Linear program reformulation ($\|\cdot\|_\star$ is dual norm):

$$\begin{aligned} \max_{u \in \mathbb{R}^{SA}} \quad & \bar{r}^{\mathsf{T}} u - \psi \|u\|_\star \\ \text{s.t.} \quad & \sum_a (\mathbf{I} - \gamma P_a^{\mathsf{T}}) u_a = p_0 \\ & u \ge \mathbf{0} \end{aligned}$$

No known VI, PI, or similar algorithms in general

Transition Function Errors

Objective:

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Focus of the remainder of tutorial

Robust Markov Decision Processes

History of Robustness for MDPs / RL

1. **1958**: Proposed to deal with imprecise MDP models in inventory management [Scarf, 1958]
2. Uncertain transition probabilities MDPs [Satia and Lave, 1973, White and Eldeib, 1994, Bagnell, 2004]
3. Competitive MDPs [Filar and Vrieze, 1997]
4. Bounded-parameter MDPs [Givan et al., 2000, Delgado et al., 2016]
5. Rectangular Robust MDPs [Iyengar, 2005b, Nilim and El Ghaoui, 2005, Le Tallec, 2007, Wiesemann et al., 2013]
6. See [Ben-Tal et al., 2009] for overview of robust optimization

Ambiguity Sets: General

Nature is constrained globally

$$\max_{\pi} \min_{P} \left\{ \mathrm{return}(\pi, P, \bar{r}) \ : \ \|P - \bar{P}\| \le \psi \right\}$$

NP-hard problem to solve e.g. [Wiesemann et al., 2013]

Ambiguity Sets: S-Rectangular

Nature is constrained for each state separately e.g. [Le Tallec, 2007]

$$\max_{\pi} \min_{P} \left\{ \mathrm{return}(\pi, P, \bar{r}) \;:\; \|P_s - \bar{P}_s\| \leq \psi_s, \, \forall s \right\}$$

Nature can see last state but not action
Polynomial time solvable; Why?

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$$\max_{\pi} \min_{P} \left\{ \mathrm{return}(\pi, P, \bar{r}) \ : \ \|P_{s,a} - \bar{P}_{s,a}\| \le \psi_{s,a},\ \forall s, a \right\}$$

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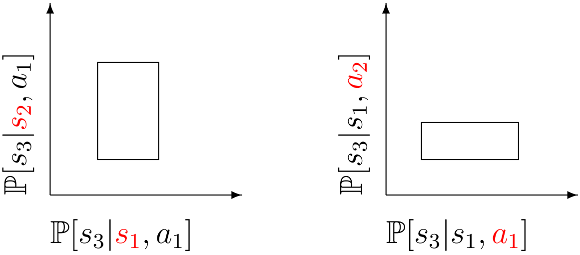
Nature can see last state and action
Polynomial time solvable; Why? Bellman Optimality

SA-Rectangular Ambiguity

Example: For each state s and action a:

$$\left\{ p_{s,a} \;:\; \| p_{s,a} - \bar{p}_{s,a} \|_1 \le \psi_{s,a} \right\} = \left\{ p_{s,a} \;:\; \sum_{s'} | p_{s,a,s'} - \bar{p}_{s,a,s'} | \le \psi_{s,a} \right\}$$

Sets are rectangles over s and a:

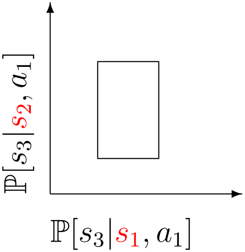


S-Rectangular Ambiguity

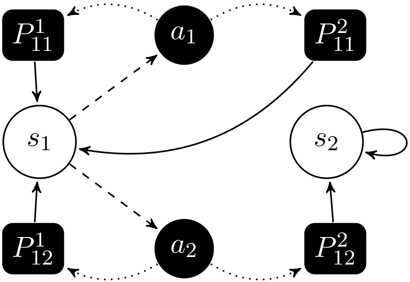
Example: For each state s:

$$\left\{ p_{s,a} \; : \; \sum_{a} \|p_{s,a} - \bar{p}_{s,a}\|_1 \leq \psi_s \right\} = \left\{ p_{s,a} \; : \; \sum_{a,s'} |p_{s,a,s'} - \bar{p}_{s,a,s'}| \leq \psi_s \right\}$$

Sets are rectangles over s only:



Robust Markov decision process



Optimal Policy Classification

Nature can be: [Iyengar, 2005a]

1. **Static**: stationary, same p in every visit to state and action
2. **Dynamic**: history-dependent, can change in every visit

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1. **Static**: stationary, same p in every visit to state and action
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Rectangularity	Static Nature	Dynamic Nature
None	H R	H R
State	H R	S R
State-Action	H R	S D

e.g. [Iyengar, 2005a, Le Tallec, 2007, Wiesemann et al., 2013]

H = history-dependent R = randomized
S = stationary / Markovian D = deterministic

Optimal Robust Value Function

Bellman optimality in MDPs:

$$v(s) = \max_a \left(r_{s,a} + \gamma \bar{p}_{s,a}^{\mathsf{T}} v \right)$$

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Robust Bellman optimality: SA-rectangular ambiguity set

$$v(s) = \max_a \min_{p \in \Delta^S} \left\{ r_{s,a} + \gamma p^{\mathsf{T}} v \;:\; \|\bar{p}_{s,a} - p\|_1 \le \psi_{s,a} \right\}$$

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Robust Bellman optimality: S-rectangular ambiguity set

$$v(s) = \max_{d \in \Delta^A} \min_{p_a \in \Delta^S} \left\{ \sum_a d(s,a)(r_{s,a} + \gamma {p_a}^{\mathsf{T}} v) \ : \ \sum_a \|\bar{p}_{s,a} - p_a\|_1 \le \psi_s \right\}$$

Solving Robust MDPs

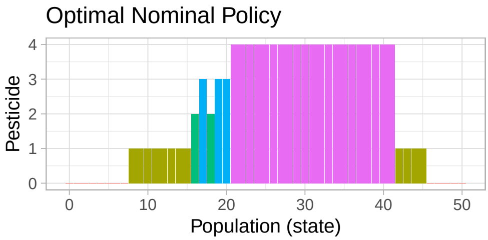
Robust Bellman operator is: e.g. [Iyengar, 2005a, Le Tallec, 2007, Wiesemann et al., 2013]

1. A contraction in L_∞ norm
2. Monotone elementwise

Therefore:

1. Value Iteration converges to the single optimal value function.
2. But naive policy iteration may loop forever [Condon, 1993]
3. No known linear programming formulation

Optimal SA Robust Policy: $\psi = 0.05$



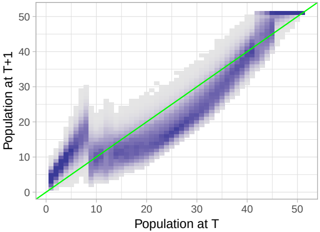
Nominal	$\$8,820$
SA-Robust	$-\$7,961$
S-Robust	$-\$7,961$

Nominal	$\$7,125$
SA-Robust	$-\$27$
S-Robust	$-\$27$

SA-Rectangular Error

Return: $\mathbf{\$7,125}$

Nominal Transitions



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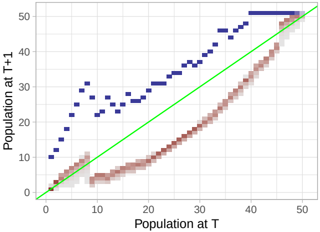
$L_1 \leq 0.05$

Noise



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Return: $\mathbf{-\$27}$

Noisy Transitions

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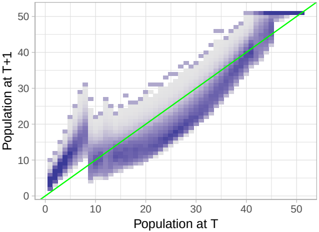
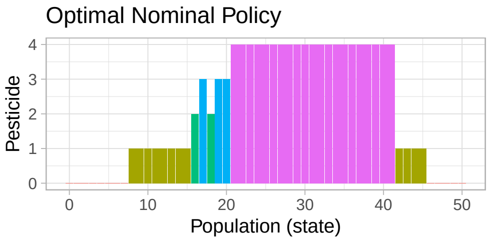


Optimal S Robust Policy: $\psi = 0.05$



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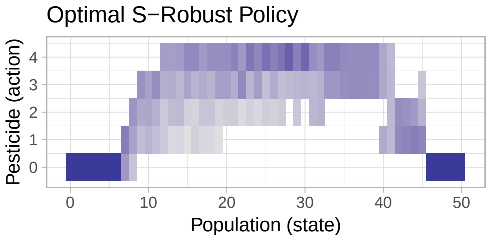
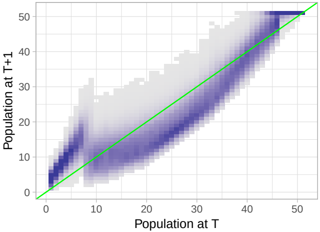


Nominal	$\$7,306$
S-Robust	$\$3,942$

S-Rectangular Error: $\psi = 0.05$

Return: $\mathbf{\$7,306}$

Nominal Transitions



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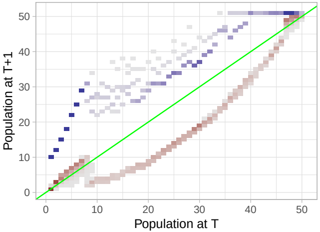

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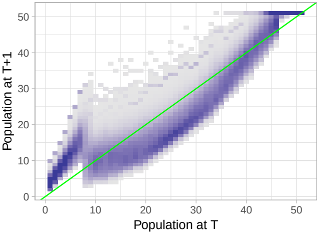
Return: $\mathbf{\$3,942}$

$=$

Noisy Transitions



Solving Robust MDPs

- **Robust Bellman Optimality**: SA-rectangular ambiguity set

$$v(s) = \max_{a} \min_{p \in \Delta^S} \left\{ r_{s,a} + p^{\mathsf{T}} v \;:\; \|\bar{p} - p\|_1 \le \psi \right\}$$

- How to solve for p?

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- How to solve for p?

- Linear programming is **polynomial time** for polyhedral sets
- Optimal policy using **value iteration** in polynomial time

- Is it really **tractable**?

Benchmarking Robust Bellman Update

Bellman update: Inventory optimization, 200 states and actions, $\psi = 0.25$

$$r_{s,a} + p^{\mathsf{T}} v$$

Time: 0.04s

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Robust Bellman update: Gurobi LP

$$\min_{p \in \Delta^S} \left\{ r_{s,a} + p^{\mathsf{T}} v \ : \ \|\bar{p} - p\|_1 \leq \psi \right\}$$

	Distance Metric	
Rectangularity	L_1 Norm	w-L_1 Norm
State-action	1.1 min	1.2 min
State	16.7 min	13.4 min

LP scales as $\geq O(n^3)$.

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State-action	1.1 min	1.2 min
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LP scales as $\geq O(n^3)$. There is a better way!

Robust Bellman Update in $O(n \log n)$

Quasi-linear time possible for many types of ambiguity sets

Metric	SA-Rectangular	S-Rectangular
L_1	e.g. [Iyengar, 2005a]	[Ho et al., 2018]
weighted L_1	[Ho et al., 2018]	[Ho et al., 2018]
L_2	[Iyengar, 2005a]	**
L_∞	e.g. [Givan et al., 2000], *	**
KL-divergence	[Nilim and El Ghaoui, 2005]	**
Bregman div	**	**

* proof in [Zhang et al., 2017], ** = unpublished result

Fast Robust Bellman Updates [Ho et al., 2018]

	Distance Metric	
Rectangularity	L_1 Norm	w-L_1 Norm
SA	$O(n \log n)$	$O(k\, n \log n)$
S	$O(n \log n)$	$O(k\, n \log n)$

Problem size: $n =$ states $\times$ actions

1. Homotopy Continuation Method: use simple structure
2. Bisection + Homotopy Method: randomized policies in combinatorial time

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SA-Rectangular Problem

Optimization: $\min_{p} \left\{ p^{\mathsf{T}} v : \|p - \bar{p}\|_1 \le \xi \right\}$

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Lift to get a linear program:

$$\begin{aligned} \min_{p,l} \quad & p^{\mathsf{T}} v \\ \text{s.t.} \quad & p_i - \bar{p}_i \le l_i \\ & \bar{p}_i - p_i \le l_i \\ & p_i \ge 0 \\ & \mathbf{1}^{\mathsf{T}} p = 1, \;\; \mathbf{1}^{\mathsf{T}} l = \xi \end{aligned}$$

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Observation: In basic solution at most <u>two</u> i: $p_i \ne 0$ and $p_i \ne \bar{p}_i$

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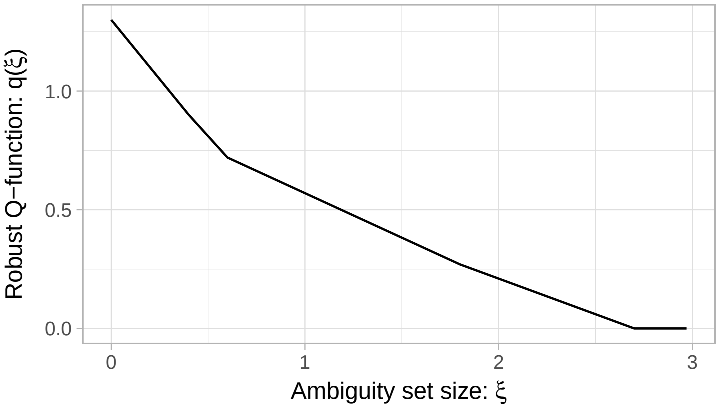
Observation: In basic solution at most <u>two</u> i: $p_i \neq 0$ and $p_i \neq \bar{p}_i$

Therefore:

1. At most S^2 basic solutions (S with no weights)
2. At most two p_i depend on budget ξ

SA-Rectangular: Homotopy Method

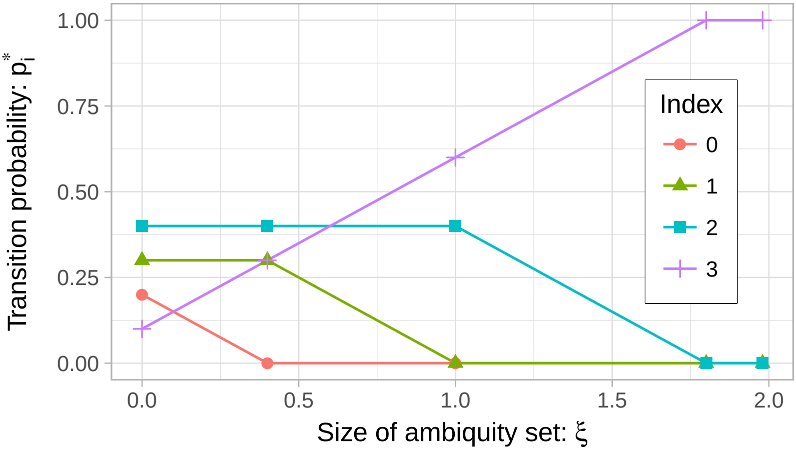
$$\min_{p \in \Delta^S} \left\{ p^\mathsf{T} v : \|p - \bar{p}\|_1 \le \xi \right\}$$



Trace optimal solution with increasing ξ

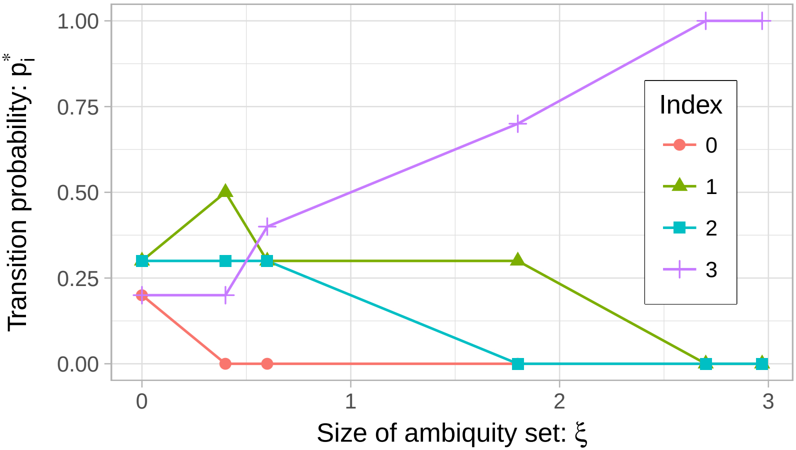
SA-Rectangular: Plain L_1

$$\bar{p} = [0.2, 0.3, 0.4, 0.1] \qquad v = [4, 3, 2, 1]$$



SA-Rectangular: Weighted L_1

$$\bar{p} = [0.2, 0.3, 0.3, 0.2] \qquad v = [2.9, 0.9, 1.5, 0.0] \qquad w = [1, 1, 2, 2]$$



S-Rectangular Optimization

Optimization problem: Linear program

$$\max_{d \in \Delta^A} \min_{p_a \in \Delta^S} \sum_a d(s,a)(r_{s,a} + \gamma\, p_a{}^\mathsf{T} v)$$

$$\text{s.t.} \quad \sum_a \|\bar{p}_{s,a} - p_a\|_1 \le \psi_s$$

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Why should it be easy to solve?

1. Use $\|\cdot\|_1$ structure from SA-rectangular formulation
2. Constraint is a sum: Decompose!

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Special S-rectangular formulation, does not work in general

Bisection to Decompose Optimization

1. Objective with $q_a(\xi) =$ SA-rectangular update:

$$\max_{d \in \Delta^A} \min_{\xi \in \mathbb{R}_+^A} \left\{ \sum_{a \in \mathcal{A}} d_a \cdot q_a(\xi_a) \,:\, \sum_{a \in \mathcal{A}} \xi_a \le \kappa \right\}$$

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2. Swap min and max (which becomes deterministic):

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3. Turn objective to constraint:

$$\min_{u \in \mathbb{R}} \min_{\xi \in \mathbb{R}_+^A} \left\{ u \,:\, \sum_{a \in \mathcal{A}} \xi_a \leq \kappa,\; \max_{a \in \mathcal{A}} q_a(\xi_a) \leq u \right\}$$

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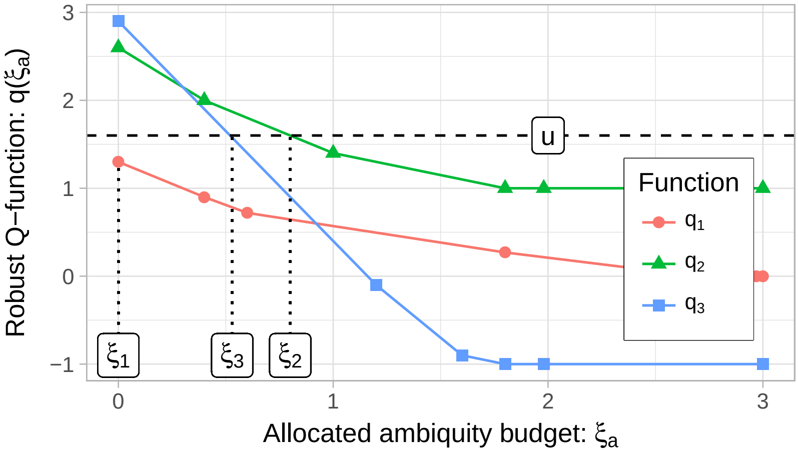
Bisect on u: $O(n \log n)$ combinatorial complexity

S-Rectangular: Bisection Method

$$\min_{u \in \mathbb{R}} \min_{\xi \in \mathbb{R}_+^A} \left\{ u : \sum_{a \in \mathcal{A}} \xi_a \le \kappa,\ \max_{a \in \mathcal{A}} q_a(\xi_a) \le u \right\}$$



Numerical Time Complexity

Timing Robust Bellman Updates: Inventory optimization, 200 states and actions, $\psi = 0.25$, Gurobi LP solver / Homotopy + Bisection

	Distance Metric	
Rectangularity	L_1 Norm	w-L_1 Norm
State-action	1.1 min / 0.6s	1.2 min / 0.8s
State	16.7 min / 0.7s	13.4 min / 1.2s

Bellman update: **0.04s**

Partial Policy Iteration: S-Rectangular RMDPs

While Bellman residual of v_k is large:

1. Policy evaluation: Compute v_k for policy π_k with precision ϵ_k (RMDP with fixed π is MDP)
2. Policy improvement: Get π_{k+1} by greedily improving policy
3. $k \leftarrow k + 1$

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3. $k \leftarrow k + 1$

Theorem: Converges fast as long as $\epsilon_{k+1} \leq \gamma^c \epsilon_k$ for $c > 1$

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Policy Iteration for Robust MDPs

- **Value Iteration**: Works as in MDPs
- Naive policy iteration may cycle forever [Condon, 1993]
- **Policy iteration** with LP as evaluation [Iyengar, 2005a]
- **Modified Robust Policy Iteration** [Kaufman and Schaefer, 2013]
- **Partial Policy Iteration**: Approximate policy evaluation [Ho et al. 2019]

Benchmarks: Scaling with States

Time in seconds, 300 second timeout, S-rectangular

	MDP	RMDP Gurobi		RMDP Bisection	
States	PI	VI	PPI	VI	PPI
12	0.00	0.36	0.01	0.00	**0.00**
36	0.00	>300	0.22	0.03	**0.00**
72	0.00	—	>300	0.13	**0.01**
108	0.00	—	—	0.31	**0.03**
144	0.01	—	—	0.60	**0.05**
180	0.02	—	—	0.93	**0.08**
216	0.03	—	—	1.38	**0.14**
252	0.04	—	—	1.84	**0.20**
288	0.06	—	—	2.46	**0.27**

Beyond Plain Rectangularity

S- and SA-rectangularity are:

[+] Computationally convenient

[−] Practically limiting

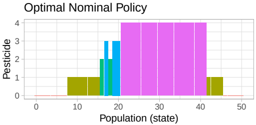
Extensions: Most based on state augmentation

- **k-rectangularity**: [Mannor et al., 2012] Upper limit on the number of deviations from nominal
- **r-rectangularity**: [Goyal and Grand-Clement, 2018]
- **other approaches**: Distributionally robust constraints [Tirinzoni et al., 2018]

Modeling Errors in RL

What Is Small Error?

Optimize $\psi = 0.0$



Evaluate

$\psi = 0$	8,850
$\psi = 0.05$	-6,725
$\psi = 0.4$	-60,171

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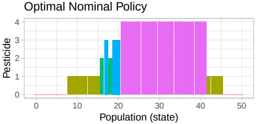

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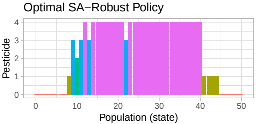

Evaluate

$\psi = 0$	7,408
$\psi = 0.05$	-25
$\psi = 0.4$	-46,256

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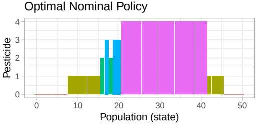
Optimize $\psi = 0.0$



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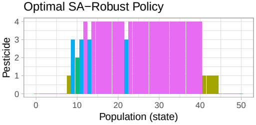
Optimize $\psi = 0.05$



Evaluate

$\psi = 0$	7,408
$\psi = 0.05$	-25
$\psi = 0.4$	-46,256

Optimize $\psi = 0.4$

Evaluate

$\psi = 0$	-622
$\psi = 0.05$	-2,485
$\psi = 0.4$	-31,613

Which ψ to optimize for?

Choosing Level Robustness (Ambiguity Set)

1. What is the right size ψ of the ambiguity set?

2. Should $\psi_{s,a}$ be the same for each state and action?

3. Why use the L_1 norm? What about L_∞, KL-divergence, Others?

4. Which rectangularity to use (if any)?

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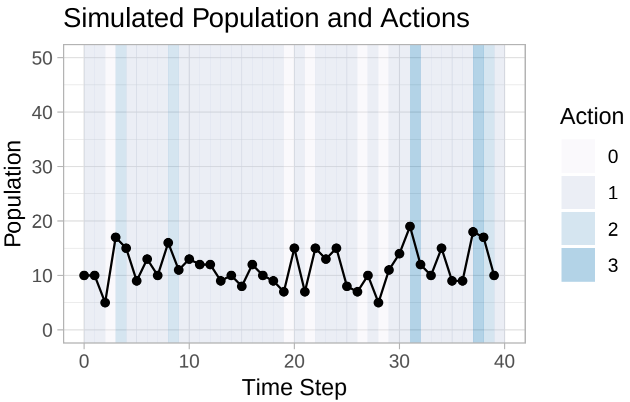
Depends on why there are errors!

Sample-efficient Batch Model-based RL

No simulator, off-policy, just compute policy (Doina's talk)

Logged data: Population (biased), actions, rewards



Model-Based Reinforcement Learning

Use Dyna-like approach: (Martha's Talk)

1. Collect transition data
2. Use ML to build transition model
3. Solve MDP model to get π
4. Deploy policy π (with crossed fingers)

The model can be wrong. Why?

Sources of Model Error

1. **Model simplification**: Value function approximation / simplified simulator [Petrik, 2012, Petrik and Subramanian, 2014, Lim and Autef, 2019]

2. **Limited data**: Not enough data; batch RL e.g. [Petrik et al., 2016, Laroche et al., 2019, Petrik and Russell, 2019]

3. **Non-stationary environment**: [Derman et al., 2019]

4. **Noisy observations**: Like POMDPs but simpler e.g. [Pattanaik et al., 2018]

Each error source requires different treatment

Robust Model-Based Reinforcement Learning

Standard approach:

1. Collect transition data
2. Use ML to build transition model
3. Solve MDP to get π
4. Deploy policy π (with crossed fingers)

Robust approach:

1. Collect transition data
2. Use ML to build transition model and confidence
3. Solve Robust MDP model to get π
4. Deploy policy π (with confidence)

Error 1: Model Simplification [Petrik and Subramanian, 2014]

State aggregation: Piece-wise constant linear value function approximation

Performance loss for $\tilde{\pi}$

$$\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi}) = \mathrm{return}(\mathrm{optimal}) - \mathrm{return}(\mathrm{approximated})$$

Loss bound [Gordon, 1995, Tsitsiklis and Van Roy, 1997]

$$\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi}) \le \frac{4\,\gamma}{(1-\gamma)^2} \min_{v \in \mathbb{R}^{\mathcal{S}}} \|v^\star - \Phi v\|_\infty$$

Robustness for State Aggregation

Transition probabilities:

	s_3	s_4
s_1	$1/4$	$3/4$
s_2	$2/3$	$1/3$

Aggregate s_1 and s_2 with weights α_1 and α_2 into s

Standard: arbitrary (wrong) α's: $\alpha_1 = 0.4, \alpha_2 = 0.6$

$$v(s) = (0.4 \cdot 1/4 + 0.6 \cdot 2/3)v(s_3) + (0.4 \cdot 3/4 + 0.6 \cdot 1/3)v(s_4)$$

Robust: adversarial α's

$$v(s) = \min_{\alpha \in \Delta^2} (\alpha_1 \cdot 1/4 + \alpha_2 \cdot 2/3)v(s_3) + (\alpha_1 \cdot 3/4 + \alpha_2 \cdot 1/3)v(s_4)$$

Reducing Performance Loss

Standard aggregation

$$\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi}) \le \frac{4\,\gamma}{(1-\gamma)^2} \min_{v \in \mathbb{R}^{\mathcal{S}}} \|v^\star - \Phi v\|_\infty$$

Uniform weights incorrect = large error

Robust aggregation

$$\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi}) \le \frac{2}{1-\gamma} \min_{v \in \mathbb{R}^{\mathcal{S}}} \|v^\star - \Phi v\|_\infty$$

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Uniform weights incorrect = large error

Robust aggregation

$$\text{return}(\pi^\star) - \text{return}(\tilde{\pi}) \le \frac{2}{1-\gamma} \min_{v \in \mathbb{R}^{\mathcal{S}}} \|v^\star - \Phi v\|_\infty$$

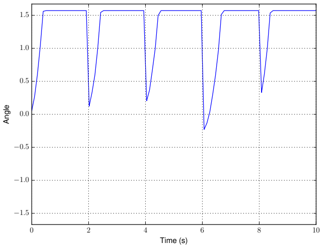
Bound constant

γ	standard	robust
0.9	360	20
0.99	36,000	200
0.999	4,000,000	2,000

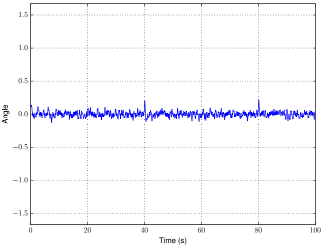
Numerical Simulation: Inverted Pendulum

Inverted pendulum with additional reward for off-balance

Regular

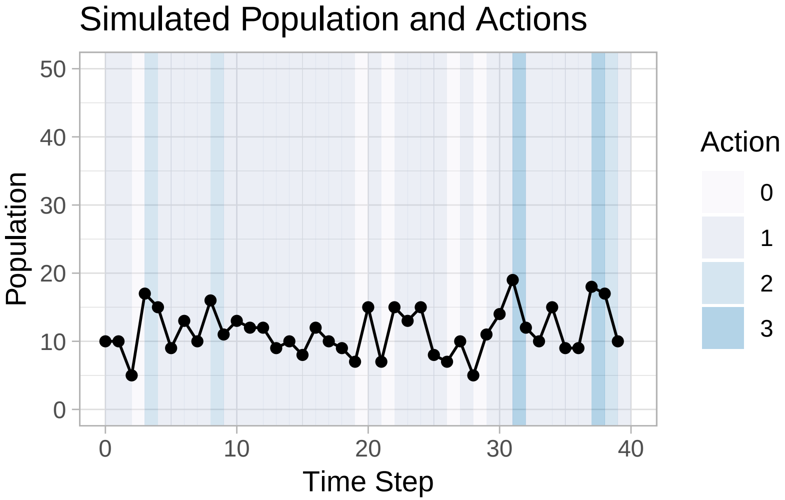


Robust



Error 2: Limited Data Availability

What is missing in this data?



Error 2: Limited Data Availability

Learn model **and confidence**: Uncertain values of P

Percentile criterion: Confidence level: δ, e.g. $\delta = 0.1$ [Delage and Mannor, 2010, Petrik and Russell, 2019]

$$\max_{\pi, y} y \text{ s.t. } \mathbf{P}_{P^\star}\left[\mathrm{return}(\pi, P^\star, r) \geq y\right] \geq 1 - \delta$$

Risk aversion: same formulation, risk-averse to epistemic uncertainty

$$\max_{\pi} \mathrm{V@R}^{1-\delta}_{P^\star}[\mathrm{return}(\pi, P^\star, r)]$$

Why this objective?

Error 2: Limited Data Availability

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Why this objective? Robust, guarantees, know when you fail

Percentile Criterion as RMDP

Percentile criterion [Delage and Mannor, 2010, Petrik and Russell, 2019]

$$\max_{\pi, y} y \text{ s.t. } \mathbf{P}_{P^\star}\left[\mathrm{return}(\pi, P^\star, r) \geq y\right] \geq 1 - \delta$$

Ambiguity set $\mathcal{P}$ designed such that:

$$\mathbf{P}_{P^\star}\left[\mathrm{return}(\pi, P^\star, r) \geq \min_{P \in \mathcal{P}} \mathrm{return}(\pi, P, \bar{r})\right] \geq 1 - \delta$$

Robustness in face of limited data

Frequentist framework

[+] Few assumptions

[+] Simple to implement

[−] Too conservative / useless?

[−] Cannot generalize

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Frequentist framework

[+] Few assumptions

[+] Simple to implement

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[−] Cannot generalize

Bayesian framework

[−] Needs priors

[+] Can use priors

[−] Computationally demanding

[+] Good generalization

Frameworks have different types of guarantees e.g. [Murphy, 2012]

Frequentist Ambiguity Set

Few samples $\longrightarrow$ large ambiguity set

Hoeffding's Ineq.: For true $p^\star$ with prob. $1-\delta$: e.g. [Weissman et al., 2003, Jaksch et al., 2010, Laroche et al., 2019, Petrik and Russell, 2019]

$$\|p^\star_{s,a} - \bar{p}_{s,a}\|_1 \le \underbrace{\sqrt{\frac{2}{n}\log\left(\frac{S A\, 2^S}{\delta}\right)}}_{\psi_{s,a}}$$

Ambiguity set for s and a:

$$\mathcal{P} = \{p \;:\; \|p - \bar{p}_{s,a}\|_1 \le \psi_{s,a}\}$$

Very conservative ... can use bootstrapping?

Bayesian Models for Robust RL

1. **Uninformative models**: Dirichlet prior for the probability distribution for each state and action. Dirichlet posterior.

$$p_{s,a} \sim \mathrm{Dirichlet}(\alpha_1, \ldots, \alpha_S)$$

2. **Informative models**: A parametric hierarchical Bayesian model. Population at time t is x_t :

$$x_{t+1} = \alpha \cdot x_t + \beta \cdot x_t^2 + \mathcal{N}(1, 10)$$

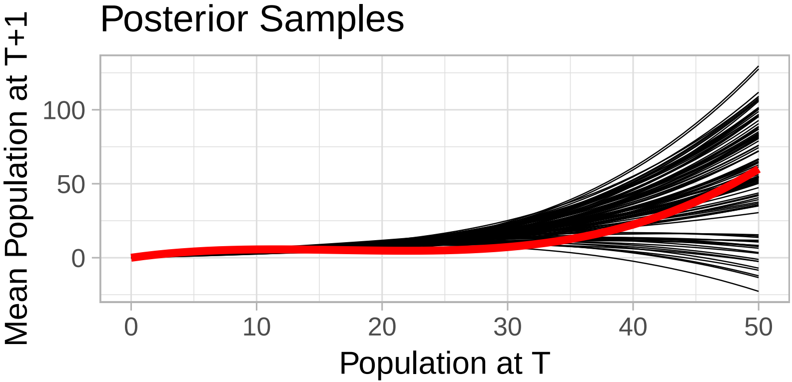
MCMC to sample from posterior over α, β
Generalize to infinite state space

Hierarchical Bayesian Models: Factored Models

MCMC using Stan, JAGS, PyMC3/4, Edward, ... to model population at time t is x_t :

$$x_{t+1} = \alpha \cdot x_t + \beta \cdot x_t^2 + \mathcal{N}(1, 10)$$

Larger population $\longrightarrow$ more uncertainty



Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3|s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3|s_0) = [10, 5, -1]$
True value: $v(s_0) = r^\mathsf{T} p^\star = 6.3$

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3|s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3|s_0) = [10, 5, -1]$
True value: $v(s_0) = r^\mathsf{T} p^\star = 6.3$

Samples: $4 \times (s_0 \to s_1)$, $6 \times (s_0 \to s_2)$, $1 \times (s_0 \to s_3)$

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3|s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3|s_0) = [10, 5, -1]$

True value: $v(s_0) = r^\mathsf{T} p^\star = 6.3$

Samples: $4 \times (s_0 \to s_1)$, $6 \times (s_0 \to s_2)$, $1 \times (s_0 \to s_3)$

1. Frequentist: $\psi = \sqrt{2/n \log(2^S/\delta)} = 0.8$

$$\hat{v}(s_0) = \min_{p:\|\bar{p}-p\|_1 \le 0.8} r^\mathsf{T} p = 2.1$$

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3|s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3|s_0) = [10, 5, -1]$

True value: $v(s_0) = r^\mathsf{T} p^\star = 6.3$

Samples: $4 \times (s_0 \to s_1)$, $6 \times (s_0 \to s_2)$, $1 \times (s_0 \to s_3)$

1. **Frequentist**: $\hat{v}(s_0) = \min_{p : \|\bar{p} - p\|_1 \le 0.8} r^\mathsf{T} p = 2.1$
2. **Bayes Credible Region**: Posterior: $p \sim \mathrm{Dirichlet}(5, 7, 1)$, samples:

$$p_1 = \begin{pmatrix} 0.2 \\ 0.7 \\ 0.1 \end{pmatrix}, p_2 = \begin{pmatrix} 0.6 \\ 0.3 \\ 0.1 \end{pmatrix}, \ldots$$

Set ψ such that 80% of p_i satisfy:

$$\|p_i - \bar{p}\|_1 \le \psi = 0.8$$

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3|s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3|s_0) = [10, 5, -1]$
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1. **Frequentist**: $\hat{v}(s_0) = \min_{p:\|\bar{p}-p\|_1 \le 0.8} r^\mathsf{T} p = 2.1$
2. **Bayes Credible Region**: $\hat{v}(s_0) = \min_{p:\|\bar{p}-p\|_1 \le 0.8} r^\mathsf{T} p = 2.1$
3. **Direct Bayes Bound**: δ-quantile of values $r^\mathsf{T} p_i$:

$$\hat{v}(s_0) = \mathrm{V@R}_{p_i}^{0.8}[r^\mathsf{T} p_i] = 5.8$$

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

Problem: $p^\star(s_1, s_2, s_3 | s_0) = [0.3, 0.5, 0.2]$, $r(s_1, s_2, s_3 | s_0) = [10, 5, -1]$
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Bayesian credible regions as ambiguity sets are too large

Samples to Ambiguity Set: Single State Value, $\delta = 0.2$

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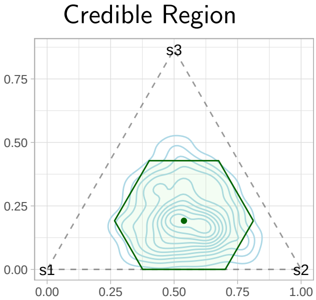
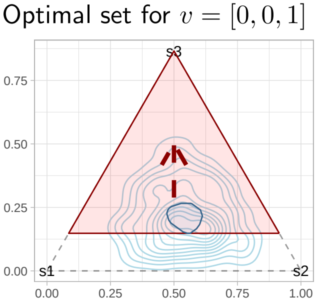
Bayesian credible regions as ambiguity sets are too large

4. **RSVF**: Approximates optimal ambiguity set $\mathcal{P}$ [Petrik and Russell, 2019]

$$\hat{v}(s_0) = \min_{p \in \mathcal{P}} r^\mathsf{T} p = 5.8$$

Optimal Bayesian Ambiguity Sets

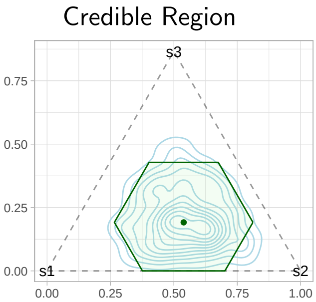


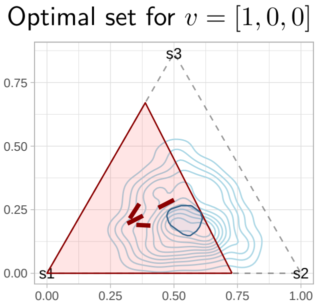


The blue set is optimal (if it exists) for all non-random v [Gupta, 2015, Petrik and Russell, 2019]

RSVF outer-approximates the optimal blue set

Optimal Bayesian Ambiguity Sets





The blue set is optimal (if it exists) for all non-random v [Gupta, 2015, Petrik and Russell, 2019]

RSVF outer-approximates the optimal blue set

Bayesian Credible Regions are Too Large: Why?

Credible region $\mathcal{P}_{s,a}$ guarantees

$$\mathbf{P}_{P^\star}\left[\min_{p\in\mathcal{P}_{s,a}} p^\mathsf{T} v \le (p^\star_{s,a})^\mathsf{T} v,\ \forall v \in \mathbb{R}^S\right] \ge 1-\delta.$$

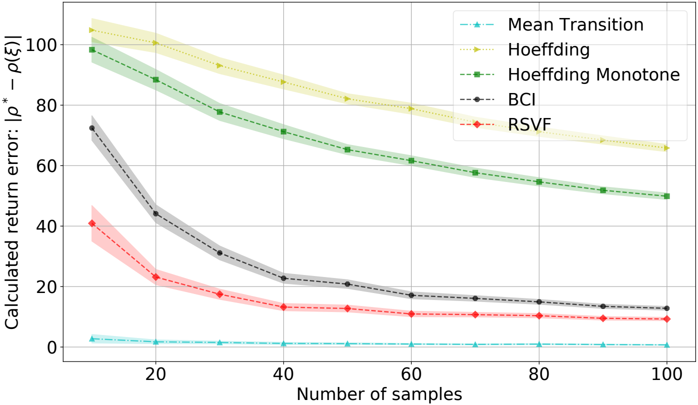
But this is **sufficient**:

$$\mathbf{P}_{P^\star}\left[\min_{p\in\mathcal{P}_{s,a}} p^\mathsf{T} v \le (p^\star_{s,a})^\mathsf{T} v\right] \ge 1-\delta,\ \forall v \in \mathbb{R}^S$$

Because v is not a random variable

How Conservative are Robustness Estimates

Population model: Gap of the lower bound. Smaller is better; 0 unachievable.



Mean: Point est.	BCI: Bayesian CI	RSVF: Near-optimal Bayesian

Other Approaches

Other Objectives

1. **Robust objective**

$$\max_{\pi} \min_{P,r} \ \mathrm{return}(\pi, P, r)$$

2. **Minimize robust regret** e.g. [Ahmed et al., 2013, Ahmed and Jaillet, 2017, Regan and Boutilier, 2009]

$$\min_{\pi} \max_{\pi^\star, P, r} \left(\mathrm{return}(\pi^\star, P, r) - \mathrm{return}(\pi, P, r)\right)$$

All NP hard optimization problems

3. **Minimize baseline regret**: Improve on a given policy π_B [Petrik et al., 2016, Kallus and Zhou, 2018]

$$\min_{\pi} \max_{P,r} \left(\mathrm{return}(\pi_B, P, r) - \mathrm{return}(\pi, P, r)\right)$$

Also NP hard optimization problem

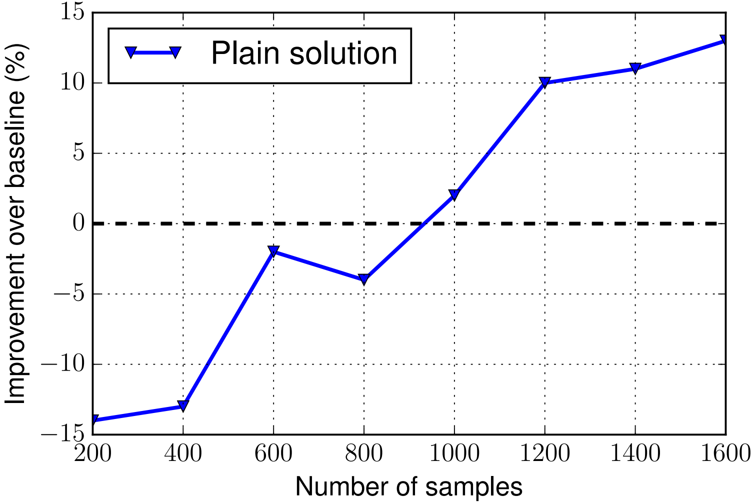
Guarantee Policy Improvement [Petrik et al., 2016]

Baseline policy π_{B}: Currently deployed, good but would like an improvement

Goal: Guarantee improvement on baseline policy

Algorithm: Minimize robust baseline regret

Solution Quality vs Samples



Safe Policy Using Robust MDP

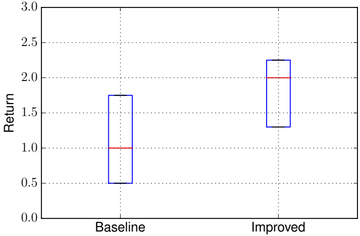
- Compute a robust policy:

$$\tilde{\pi} \leftarrow \arg\max_{\pi} \min_{\xi} \operatorname{return}(\pi, \xi)$$

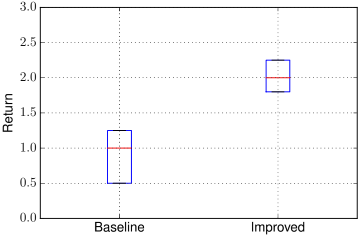
- Accept $\tilde{\pi}$ if outperforms π_{B} with prob $1 - \delta$:

$$\min_{\xi} \operatorname{return}(\tilde{\pi}, \xi) \geq \max_{\xi} \operatorname{return}(\pi_{\mathsf{B}}, \xi)$$

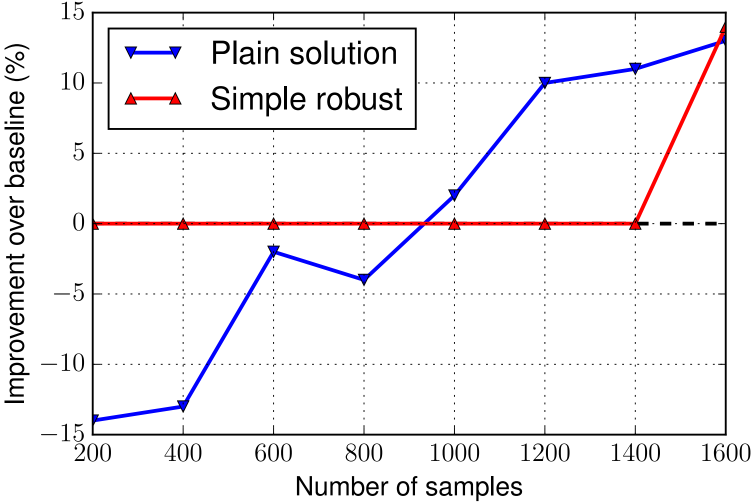
Reject



Accept



Benchmark: Robust Solution



Limitation of Simple Robustness: Improving Commute

Usual commute	Better commute?
Interstate: 20 min *Bridge*: 10–30 min	*Local road*: 10 min *Bridge*: 10–30 min
Total: 30–50 min	Total: 20–40 min

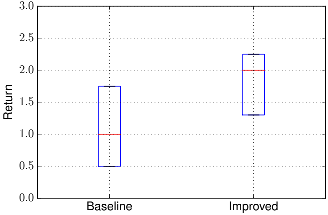
Reject: 40 min $>$ 30 min

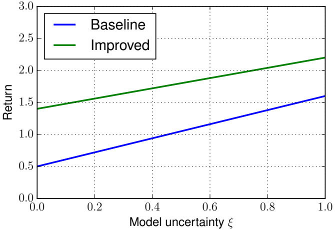
Minimizing Robust Baseline Regret

- **Minimize robust baseline regret**

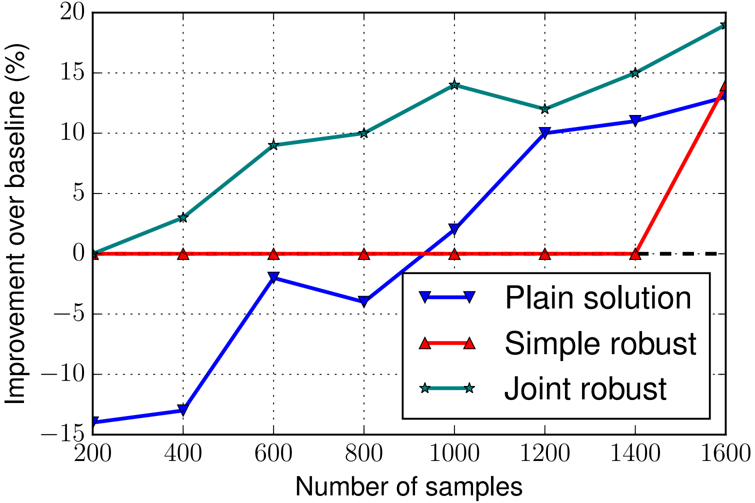
$$\min_{\pi} \max_{\xi} \Big(\mathrm{return}(\pi_{\mathsf{B}}, \xi) - \mathrm{return}(\pi, \xi) \Big)$$

- Correlation between impacts of robustness





Benchmark: Minimizing Robust Baseline Regret



Minimizing Robust Baseline Regret

- Optimal stationary policy may have to be **randomized**
- Arbitrary optimality gap for deterministic policies
- Computing optimal deterministic policy is **NP hard**

$$\max_{\pi} \min_{\xi} \Big(\mathrm{return}(\pi, \xi) - \mathrm{return}(\pi_{\mathsf{B}}, \xi) \Big)$$

- Even computing nature response in **NP hard**

$$\min_{\xi} \Big(\mathrm{return}(\pi, \xi) - \mathrm{return}(\pi_{\mathsf{B}}, \xi) \Big)$$

- NP-hard even with **rectangular** uncertainty

Performance Guarantees

Model error:

$$\|p^\star_{s,a} - \bar{p}_{s,a}\|_1 \le \underbrace{\sqrt{\frac{2}{n}\log\left(\frac{S\,A\,2^S}{\delta}\right)}}_{e(s,a)}$$

Classic performance loss:

$$\underbrace{\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi})}_{\text{Policy loss}} \le C \; \underbrace{\max_\pi \|e_\pi\|_\infty}_{L_\infty\,\text{norm}}$$

Performance loss (regret) for robust solution:

$$\underbrace{\mathrm{return}(\pi^\star) - \mathrm{return}(\tilde{\pi})}_{\text{Policy loss}} \le \min\Big\{\; C\; \underbrace{\|e_{\pi^\star}\|_{1,u^\star}}_{L_1\ \text{norm}},\; \underbrace{\mathrm{return}(\pi^\star) - \mathrm{return}(\pi_{\mathsf{B}})}_{\text{Baseline loss}} \;\Big\}$$

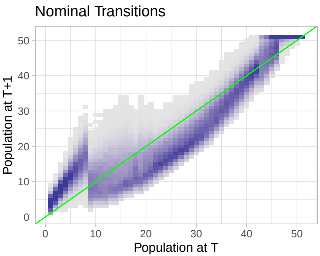
Summary

Robustness is Important In RL

1. Learning without a simulator:
   - Insufficient data set size
   - How to test a policy? No cross-validation

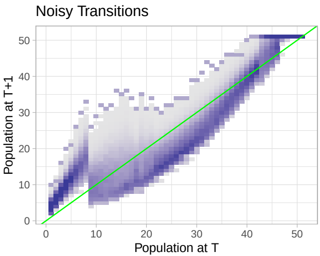
2. High cost of failure (bad policy)

Return: $\$8,820$



$\Rightarrow$

Return: $-\$6,725$



RL with Robust MDPs

"Model-based approach to reliable off-policy sample-efficient tabular RL by learning models and confidence"

- **RMDPs are a convenient model for robustness**
  - Tractable methods with rectangular sets
  - Provide strong guarantees
- **Learn a model and its confidence**
  - Source of error matters
  - Promising methods for small data
- Many model-free methods too e.g. [Thomas et al., 2015, Pinto et al., 2017, Pattanaik et al., 2018]

Important Research Directions

1. **Scalability** [Tamar et al., 2014]
   - Value function approximation: Deep learning et al
   - How to preserve some sort of guarantees?
2. **Relaxing rectangularity**
   - Crucial in reducing unnecessary conservativeness
   - Tractability?
3. **Applications**
   - Understand the real impact and limitations of the techniques
4. Code: `http://github.com/marekpetrik/craam2`, well-tested, examples, but unstable, pre-alpha

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