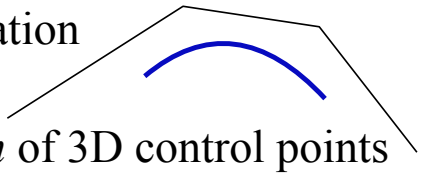
CS770/870 Spring 2017 Surface Generation

• Primary resources used in preparing these notes:

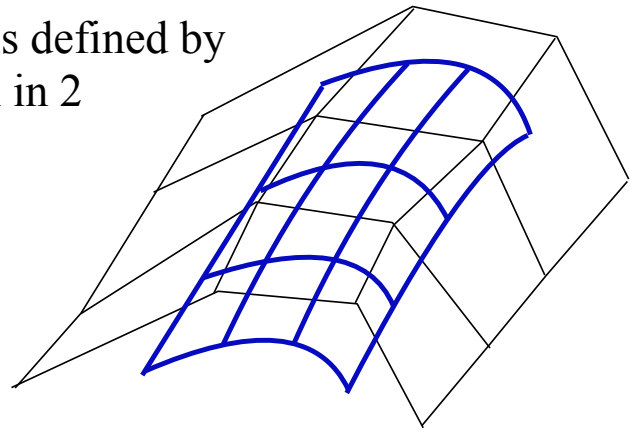
Foley, van Dam, Feiner, Hughes, Phillips, **Introduction to Computer Graphics**, Addison-Wesley, 1993.

Overview

- Can define 3D curve from (nonplanar) 3D control points
  - Curve defined by parametric equation in *t* (here, a B-spline)



- Can define a 2D surface from a *mesh* of 3D control points
  - Each surface point is defined by parametric equation in 2 variables (s,t)



Parametric Bicubic Surfaces

- Generalize parametric bicubic curves
  - curve(s) = PMS where M is blending function coefficient matrix, P = [P_0, P_1, P_2, P_3] and $S = \begin{vmatrix} 1 \\ s \\ s^2 \\ s^3 \end{vmatrix}$
- Suppose now that each point in P is a function of t:
  - surface(s,t) = [$P_0(t)$, $P_1(t)$, $P_2(t)$, $P_3(t)$]MS
  - for $t=t_1$, $S(s,t_1)$ is a curve because $P(t_1)$ is constant
  - Let $t_2=t_1+dt$; $P(t_2)$ is also a curve that is similar to $P(t_1)$ if dt is small; $P(t_2)$ is similar to $P(t_2+dt)$, etc.
- If $P_i(t)$ are cubics, $P_i(t) = P_iMT$, where $P_i=[p_{i0}\ \ p_{i1}\ \ p_{i2}\ \ p_{i3}]$
  - $(P_iMT)^T = T^TM^TP_i^T$ and $S(s,t) = T^TM^TPMS$ where $P=[p_{ij}]$ for i,j = 0,1,2,3

B-Spline Surface

- B-spline patches:
  - $B(s,t) = T^TM^T_BPM_BS$
    where $S = [1, s, s^2, s^3]^T$ (column vector)
    $T^T = [1, t, t^2, t^3]$ (row vector)

$$M_B = \frac{1}{6}\begin{vmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{vmatrix} \qquad P = \begin{vmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ p_{30} & p_{31} & p_{32} & p_{33} \end{vmatrix}$$

  - Each $p_{ij} = [x_{ij}, y_{ij}, z_{ij}]$, so
    $X(s,t) = T^TM^T_BXM_BS$, where X = array of x_{ij} values
    $Y(s,t) = T^TM^T_BYM_BS$, where Y = array of y_{ij} values
    $Z(s,t) = T^TM^T_BZM_BS$, where Z = array of y_{ij} values

Surface Normals

- Normals to parametric bicubic surfaces are easy:
- First get tangents at a point on surface

$$\partial B(s,t)/\partial s = \partial/\partial s(T^T M^T_B P M_B S)$$
$$= T^T M^T_B P M_B \partial S/\partial s$$
$$= T^T M^T_B P M_B \partial/\partial s[1,\ s,\ s^2,\ s^3]$$
$$= T^T M^T_B P M_B [0,\ 1,\ 2s,\ 3s^2]$$
$$\partial B(s,t)/\partial t = \partial/\partial t(T^T M^T_B P M_B S)$$
$$= [0,\ 1,\ 2t,\ 3t^2] M^T_B P M_B S$$

- Then take cross product

$$\partial B/\partial s \times \partial B/\partial t = [\ y_s z_t - y_t z_s,\ \ z_s x_t - z_t x_s,\ x_s y_t - x_t y_s\ ]$$

Line Display of Bicubic Surfaces

- Curved line display is straightforward (from Foley et al.)

```
// X(s,t), Y(s,t), Z(s,t) implement M^T_B P M_B products
void drawSurf( float[4][4][3] P, int ns, int nt, int n)
   dCurve = 1/n // step size for curve drawing
   dS = 1/ns    // s parameter step size
   dT = 1/nt    // t parameter step size
   for ( i = 0; i < ns; i++ ) // curves with constant s
      s = i * dS
      move( X(s,0), Y(s,0), Z(s,0) )
      for ( j=0; j<n; j++ )  //draw curve in s dir
         t = j * dT
         line( X(s,t), Y(s,t), Z(s,t) )
   for ( i = 0; i < ns; i++ ) // curves with constant t
      t = i * dT
      move( X(0,t), Y(0,t), Z(0,t) )
      for ( j=0; j<n; j++ )  //draw curve in s dir
         s = j * dS
         line( X(s,t), Y(s,t), Z(s,t) )
```

Matrix approach

- For simplicity of presentation Foley et al. implement the X, Y, and Z functions independently as algebraic expressions, which means re-evaluating blending function calculations.
- Can use a matrix approach and have more shared code:

```
for each B-spline "patch"
  build 4x4 Px=MTBXMB, Py=MTBYMB, and Pz=MTBZMB
  for each s in [0,1] by ds
    build S matrix
    for each t in [0,1] by dt
      build T matrix
      X(s,t)=TTPxS; Y(s,t)=TTPyS; Z(s,t)=TTPzS;
```


Triangle Generation

- For surface (vs. line) drawing, often have larger delta
- Line drawing or matrix approach can get 2D point array
  - Triangulate between neighbors
- Modify either algorithm to save previous point in inner *for* loop and entire "row" of points in previous row, so can generate triangles as generating points.
- GLSL Geometry Shader is designed specifically for surface generation of this sort.
  - Geometry shader inserts new vertices into the shader pipeline
  - Matrix approach is natural and should be more efficient.