

CS770/870 Spring 2017

Radiosity

Greenberg, SIGGRAPH '86 Tutorial

Spencer, SIGGRAPH '93 Slide Set, siggraph.org/education/materials/HyperGraph/radiosity/radiosity.htm

Watt, *3D Computer Graphics -- Third Edition*, Addison-Wesley, 2000.

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Preview

- Indirect light models
- Brief radiosity overview
- Radiosity details
 - bidirectional reflectance
 - radiosity equation
 - radiosity approximation
- Implementation details
 - hemicube

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Models for Indirect Light

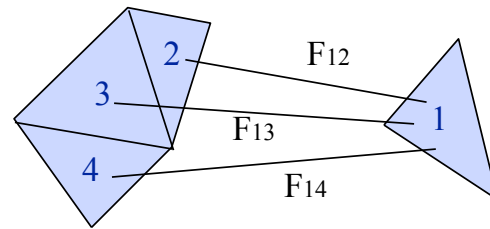
- Interreflection of objects in scene
 - light reflected from each object can affect brightness of other objects
- Ray tracing models indirect *specular* reflection
 - following rays along direction of principal reflection models interreflection of shiny surfaces
- Radiosity models indirect *diffuse* reflection
 - originally developed to model heat transfer
 - first applied to computer graphics by Don Greenberg and students at Cornell in early 80's



Cornell Program in
Computer Graphics

Radiosity Algorithm — Setup

- Light sources and objects treated alike
- Partition all objects into small *patches*
- Find interreflection between each patch pair — called *form factors* — includes visibility
- F_{12} = % light leaving patch 1 that reaches patch 2
- Patch 2 reflects R_2 % of light that lands on it, so
- Patch 2 reflects $R_2 * F_{12}$ light shot from Patch 1



Bidirectional Reflectance

- Relationship of light energy arriving at a point, P_i , to the light leaving that point

$$\rho = I_r / E_i$$

where I_r is reflected intensity per unit area

and E_i is incoming energy per unit area at P_i

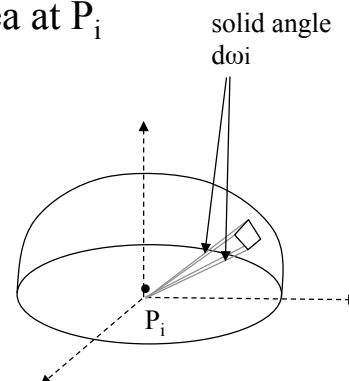
- What is the incoming energy, E_i ?

$$E_i = f(I_i, d\omega_i), \text{ some function}$$

where I_i is the incoming light *intensity*

and $d\omega_i$ is solid angle through which intensity arrives

- What is the f ? What is I_i ?



What is Radiosity?

- Radiosity (B): energy per unit area leaving a surface per unit time; it is the sum of:
 - energy *emitted* at that point
 - energy *reflected* at that point

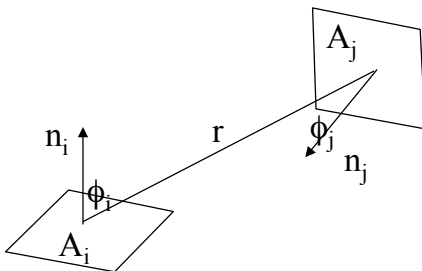
- Radiosity of a patch i whose area is dA_i :

$$B_i dA_i = E_i dA_i + R_i \int_j B_j F_{ji} dA_j$$

- E_i is energy emitted by patch i per unit area
- R_i is the % of incident energy that is reflected (per wavelength)
- F_{ji} is the *form factor* that defines the relationship between patch j and patch i .

Form Factor

- What is the *form factor* from patch i to patch j ?
 - Assume all surfaces are perfect Lambertian diffuse reflectors
 - I.e., all incident energy is reflected *equally* in all directions.
 - F_{ij} is the percent of energy leaving patch i that actually hits j
 - *occlusion*: if another patch is between i and j , $F_{ij} = 0$; vis_{ij}
 - also depends on the relative orientation of the 2 patches:



What does it mean?

$$f_{ij}(dA_i, dA_j) = \frac{\cos \phi_i \cos \phi_j}{\pi r^2} dA_i dA_j$$

$$\text{So, } f_{ij}(dA_i, A_j) = dA_i \frac{\cos \phi_i}{\pi r^2} \int_{A_j} \cos \phi_j dA_j$$

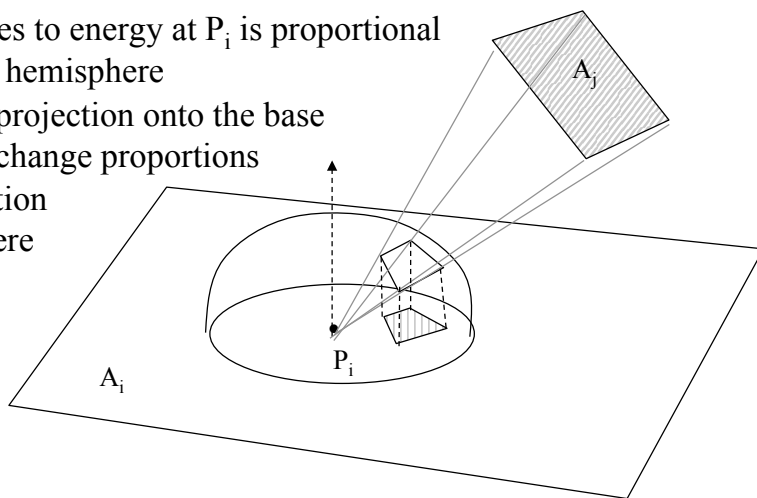
$$\text{and } f_{ij}(A_i, A_j) = \int_{A_i} \int_{A_j} \frac{\cos \phi_i \cos \phi_j}{\pi r^2} dA_j dA_i$$

To make unitless, divide by A_i ; also add occlusion test

$$F_{ij}(A_i, A_j) = \frac{vis_{ij}}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \phi_i \cos \phi_j}{\pi r^2} dA_j dA_i$$

Geometric Interpretation of Form Factor Integral

- Light is reflected by patch i equally in all directions
 - represent reflection by hemisphere with $r = 1$ around patch center
 - all energy arriving at P_i also comes through the hemisphere
 - project A_j onto hemisphere
 - contribution of other patches to energy at P_i is proportional to their projection onto the hemisphere
 - projecting the hemisphere projection onto the base of the hemisphere doesn't change proportions
 - form factor equals the fraction of the base of the hemisphere covered by the projection.



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Calculating Form Factors

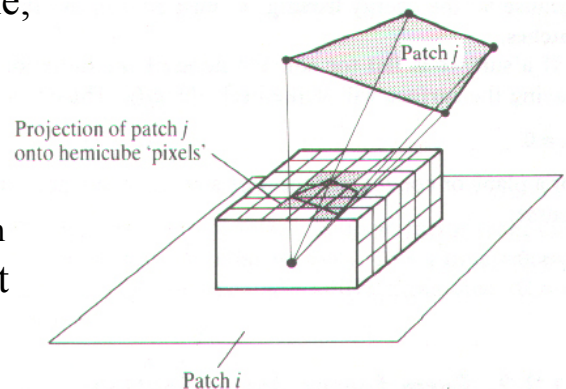
- Greenberg's group at Cornell borrowed radiosity from the heat transfer field
 - image synthesis, however, needs much higher patch resolution than heat transfer modeling
 - their principal initial contribution was a tractable algorithm for computing form factors
 - the hemicube is a rectangular approximation to the hemisphere that is placed over the center of each patch

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Hemicube

- Each face of the hemicube is projected onto the hemicube base
 - Compute *delta form factors* for each hemicube “pixel” versus the hemicube base; analytic approach yields a closed form!
 - Given hemicube size and resolution, delta form factors are the same for all hemicubes for all time; they are pre-computed and saved
 - Project each patch onto hemicube
 - form factor for the patch is the sum of the delta form factors of all hemicube pixels covered by the patch
 - A major computational component



Radiosity Equation Revisited

$$B_i dA_i = E_i dA_i + R_i \int_j B_j F_{ji} dA_j$$

But, there is a reciprocity between F_{ji} and F_{ij} : $F_{ij} A_i = F_{ji} A_j$

$$\text{So, } B_i dA_i = E_i dA_i + R_i \int_j B_j F_{ij} dA_i$$

$$\text{Dividing by } dA_i \text{ yields: } B_i = E_i + R_i \int_j B_j F_{ij}$$

$$\text{This can be approximated by: } B_i = E_i + R_i \sum_j B_j F_{ij}$$

The E_i are known, so rewrite as :

$$B_i - R_i \sum_j B_j F_{ij} = E_i$$

Radiosity Solution

- There are n radiosity equations, one for patch
 - Leads to n simultaneous linear equations to be solved

$$\begin{bmatrix} 1 - R_1 F_{11} & -R_1 F_{12} & \cdots & -R_1 F_{1n} \\ -R_2 F_{21} & 1 - R_2 F_{22} & \cdots & -R_2 F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -R_n F_{n1} & -R_n F_{n2} & \cdots & 1 - R_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

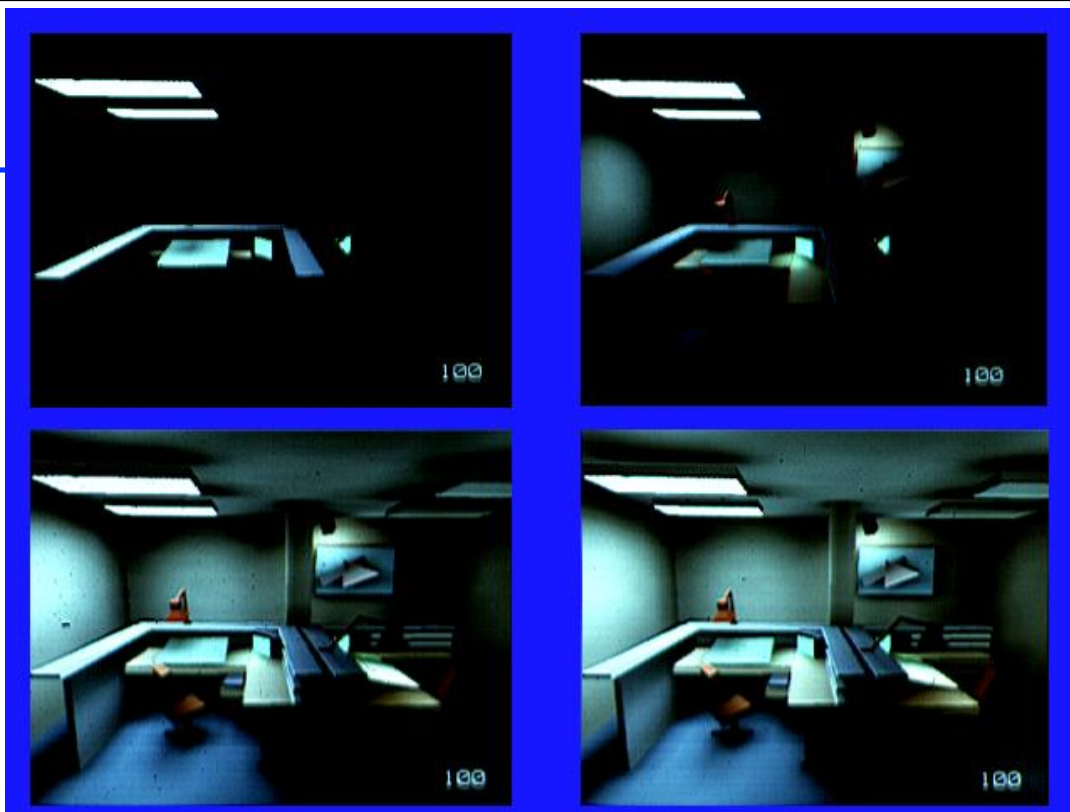
- E_i are non-zero only for light patches, R_i are known
- Can solve with something like *Gauss-Seidel* method
 - guaranteed to converge rapidly for this kind of data
 - in effect, it looks at each patch, *gathers* current contributions from all other patches to update this patch and iterates
 - can think of it as *breadth-first* approach

Progressive Refinement: Shooting

- In 1988, Cornell group developed a progressive refinement solution that rendered while computing
 - the *gather* approach of *Gauss-Seidel* was much too slow
 - they changed the focus to a *shooting* one
- Choose a patch
 - determine its contribution to all other patches
 - update the image
 - iterate until “done”
- Key difference is that can choose patches adaptively:
 - initially want to choose light patches
 - in general, choose a patch that will have most effect

Adaptive Shooting

- Associate *unshot* light with each patch — initially set to light *emitted* by the patch
 - for each patch with unshot light, *shoot* that light to all other patches (using form factor)
 - each patch *gathers* light shot to it — what it gathers at one stage is added to its *unshot* light for next stage
- At each step, pick the patch with the most unshot light
- Repeat until all patches have very little unshot light
- Add a global ambient light term
 - start with an estimate based on total light in scene
 - reduce the ambient term at each iteration based on unshot light



Siggraph Tutorial on Radiosity

Radiosity Algorithm

```
Partition all objects into patches
Compute form factors between patches
globalAmbient = estimateAmbient()
maxPatch = findMaxUnshotPatch()
while ( maxPatch.unshot > minimum )
    foreach patch (except this one)
        update B with this unshot light
    reduce globalAmbient
    rerender image
    maxPatch = findMaxUnshotPatch()
```

Radiosity Algorithm Updates

- The entire algorithm does not always need to be executed. What happens if something change:
 - Viewpoint
 - None of radiosity calculations change; only need to redraw!
 - Light intensity/colors
 - No form factors change, so only need to re-shoot the light
 - Geometry changes
 - requires form factor re-computation; this is the slowest part

Radiosity Characteristics

- Strengths
 - Highly realistic diffuse reflection, including soft shadows, and area light sources
- Weakness: can't easily address specular inter-reflection
- Artifact issues
 - Hemicube-based artifacts
 - aliasing can occur because of the regular subdivision nature of the hemicube
 - aliasing can occur where a light patch is close to other patches
 - all issues can be ameliorated with higher resolution, and higher cost
 - Interpolation issues
 - Meshing issues

Review

- Radiosity
 - bidirectional reflectance
 - radiosity equation
 - radiosity approximation
- Implementation details
 - hemicube
 - gather approach
 - shooting approach